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YAWING AND BALLOTING MOTION OF A PROJECTILE IN THE BORE OF A GUN WITH APPLICATION TO GUN TUBE DAMAGE

Evan Harris Walker

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New basic Equations of motion are derived to describe in bore shell yawing and balloting motion. It is shown that the prior equations of Reno and Thomas are not satisfactory. These equations are used to derive expressions for the growth of balloting motion. Application of these equations to a mechanical failure of a shell in an 8" howitzer XM201 test program shows the new treatment accounts for the observed tube damage. This approach provides a basis for calculating causes and probability of occurances of severe balloting.

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SUMMARY

New equations for the yawing and balloting motion of a projectile in the bore of a gun have been derived. These equations show that the previous theories of Reno and Thomas omitted important forces acting to cause in some cases severe buildup of balloting motion. These equations also show the prior treatment to be unsatisfactory even where severe balloting does not occur. The new equations will be used to compute the details of shell in bore motion in a subsequent report.

In the present report, the general equations for shell in bore motion have been used to derive equations describing specifically the growth of balloting motion to show how balloting can build to severe levels causing tube damage. These equations also make possible a calculation of the occurrence probability for severe balloting and allow one to determine important factors contributing to mechanical failures and prematures.

Balloting motion is also important in causing severe shell engraving and wearing of gun tubes. Thus control of balloting motion should significantly increase the service life of gun tubes.

The theoretical equations have been applied to the calculation of balloting in the XM201 8" Howitzer tube. Agreement between observed damage and the results of computations is obtained in these calculations.

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I. INTRODUCTION

This report stems from an investigation of a shell failure that occurred while testing the 8-inch howitzer tube XM201. The characteristics of the damage to the tube indicated that the shell, before breaking up, balloted with sufficient violence to cause impact shearing of lands in three separate sections of the tube. This tube damage will be considered in more detail below.

A consideration of the Reno[1*] and Thomas[2] theories of shell motion in a gun tube and Gay's[3] application of those theories indicates that they do not predict conditions that would lead to violent motion of the shell. Gay found that these theories predict that the yawing motion of the shell will bring the shell bourrelet into contact with the lands (if not initially resting against the tube wall) early in the motion of the shell and that yawing motion will be rapidly damped. The speed of the bourrelet approaching the lands is never large. For a coefficient of restitution e = 1, a bourrelet clearance of 0.02 inch (a large clearance) and using Reno's equations (which give slightly larger values than Thomas'), Gay's calculations give (see his Figure 4) 60.0 cm/sec (1.97 ft/sec). This is not a significant speed; the 8-inch shell which broke up in the tube had passed a 7-foot drop test (velocity at impact, 647 cm/sec) and impact against the lands of the gun tube at such speeds will not cause damage.

Other evidence cited by Gay would also suggest that severe balloting has been a recurring problem. Gay shows (his Figure 12) a 105mm shell in flight beside the broken-off fuze. The engraving noted on 8-inch shell at the bourrelet also indicates that balloting frequently exceeds that predicted by the theoretical treatments.

One should expect, however, that under appropriate conditions an instability in the motion of a shell driven by a force acting behind the shell's center of gravity (c.g.) should occur. Such a motion should resemble "chatter" that occurs in other mechanical systems. The following analysis shows that the treatment given by Thomas requires modification to incorporate the forces acting on the shell at the rotating band and a minor correction to the form of the equations giving constants of the motion. The treatment given here is also somewhat more general in that it incorporates certain significant effects on the shell motion due to the elastic nature of the shell.

By taking the time average contribution of the impact to the lateral motion of a shell, it is possible to derive closed form expressions for the growth of balloting and show what parameters of this motion favor its occurrence.

References are listed on page 38.

II. THE EQUATIONS OF MOTION

We follow the procedure after Goldstein[4] for the similar problem of the motion of a symmetric top supported at one point and acted on by gravity. The Lagrangian procedure is used to obtain the equations of motion. Since the shell may be taken to be symmetric about its axis (also assumed by Reno and by Thomas), the kinetic energy can be written as

$$T = \frac{1}{2} I(\omega_y^2 + \omega_z^2) + \frac{1}{2} A\omega_x^2, \tag{1}$$

where ω_x, ω_y, ω_z are the angular velocities about the respective axis and I is given by

$$I = B + m\ell^2, \tag{2}$$

where A and B are the axial and transverse moments of inertia of the shell about the c.g., m is the mass of the shell, and ℓ is the distance from the point C to the c.g. as indicated in Figure 1. In terms of the Euler angles, this is

$$T = \frac{1}{2} I(\dot{\delta}^2 + \dot{\phi}^2 \sin^2\delta) + \frac{1}{2} A(\dot{\psi} + \dot{\phi} \cos \delta)^2. \tag{3}$$

In a coordinate system moving with the point C and subject to the acceleration $\ddot{s}$ of the projectile, there exists an instantaneous potential field for the projectile given by

$$V = m\ddot{s}\, \ell \cos \delta. \tag{4}$$

The Lagrangian is

$$L = T - V, \tag{5}$$

and the Lagrangian equation is

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i; \quad i = 1,2,3 \tag{6}$$

where

$$\begin{aligned} q_1 &= \delta \\ q_2 &= \phi \\ q_3 &= \psi \end{aligned} \tag{7}$$

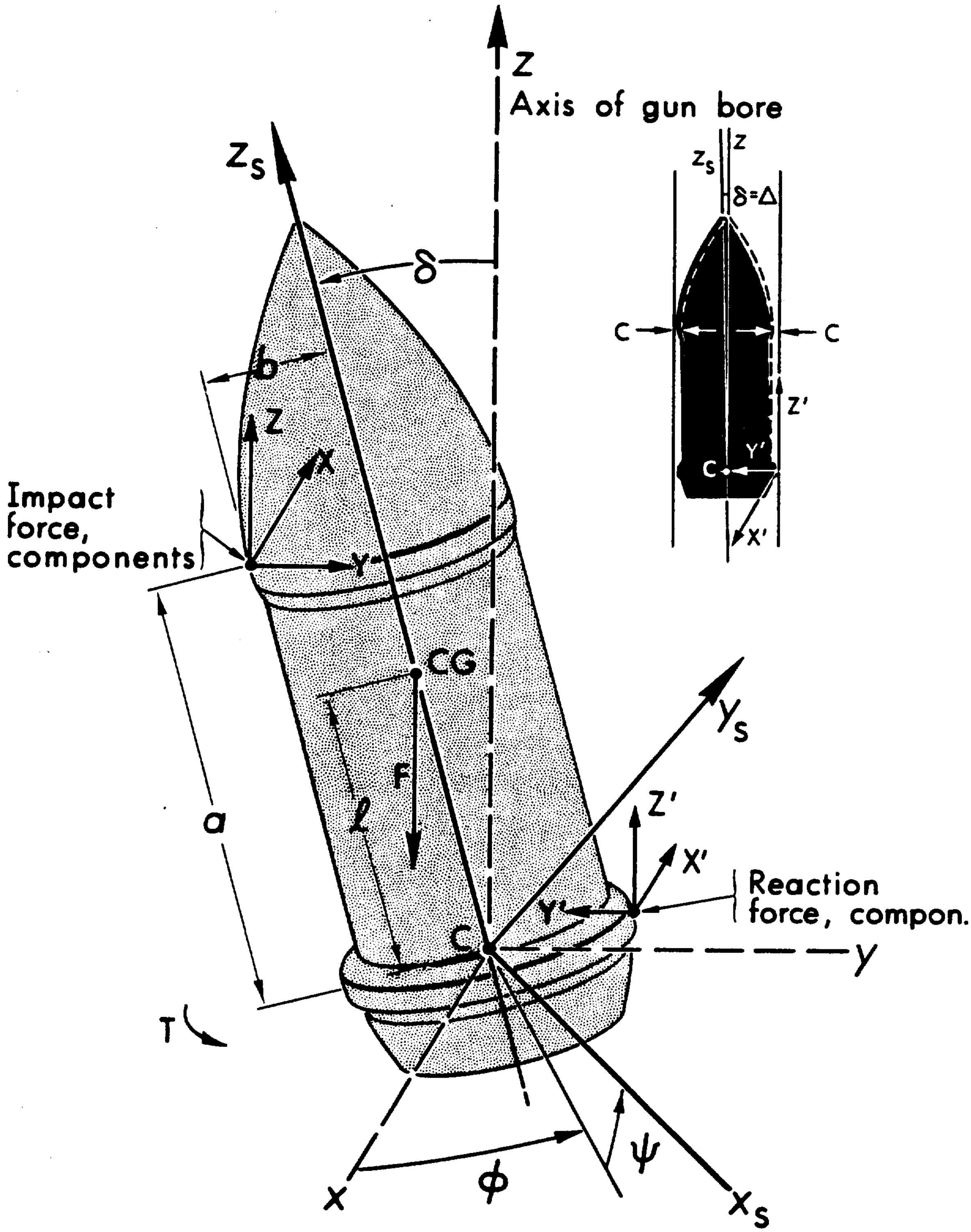

Figure 1. Euler's angles, δ, φ, ψ, specifying the orientation of a symmetrical shell about a point C at the center of the rotating band on the shell axis. The accelerating force F, and the components of the impact forces acting on the bourrelet X, Y, Z, and the reaction forces on rotating band X', Y', Z' are indicated together with the driving torque on the rotating band due to the rifling of the tube. The yaw angle δ is greatly exaggerated. Also shown are the dimensions a, the distance between the rotating band and bourrelet, ℓ, the distance from C to the center of gravity, CG, and b, the radius at the bourrelet. The insert shows the clearance c at the bourrelet and the reaction forces at the rotating band.

and the Q_i are the generalized forces in the respective directions;

$$Q_1 d\delta = [Y(a \cos \delta - b \sin \delta) - Z(a \sin \delta + b \cos \delta) + Z'r] d\delta \qquad (8)$$

where the moment arm of Y' has been taken to be zero. If the rotating band does not deform into a section of a sphere of radius r but remains a plane, $Z'r$ should be replaced by $Y'b' \sin \delta + Z'b' \cos \delta$ where b' is the distance between c and the point of "center of contact" between the rotating band and the bore ("center of contact" refers to the point at which the effective or equivalent of the forces acting on the rotating band originates). In the ϕ direction

$$Q_2 d\phi = [- X(a \sin \delta + b \cos \delta) + X'r \cos \delta] d\phi \qquad (9)$$

and

$$Q_3 d\psi = [- Xb + X'r + T] d\psi, \qquad (10)$$

where T is the torque on the rotating bands as such, not arising from impact action and reaction forces.

Equations (3) through (10) yield

$$\begin{aligned} I\ddot{\delta} - I\dot{\phi}^2 \sin \delta \cos \delta + A(\dot{\psi} + \dot{\phi} \cos \delta)\dot{\phi} \sin \delta - m\ddot{s}\, \ell \sin \delta &= Q_1 \\ &= (a \cos \delta - b \sin \delta)Y - (a \sin \delta + b \cos \delta)Z + rZ' \end{aligned} \qquad (11)$$

$$\begin{aligned} \frac{d}{dt} [I\dot{\phi} \sin^2 \delta + A(\dot{\psi} + \dot{\phi} \cos \delta)\cos \delta] &= Q_2 \\ &= -(a \sin \delta + b \cos \delta)X + r \cos \delta\, X' + T \cos \delta \end{aligned} \qquad (12)$$

$$\frac{d}{dt} [A(\dot{\psi} + \dot{\phi} \cos \delta)] = Q_3 = - bX + rX' + T. \qquad (13)$$

Before proceeding, it will be appropriate to compare Equations (11-13) with the expressions derived by Thomas. According to the assumptions made by Thomas, when $\delta < \Delta$, the forces Q_1, Q_2, and Q_3 were set equal to zero. Under these assumptions, Equations (11-13) yield

$$I\ddot{\delta} - I\dot{\phi}^2 \sin \delta \cos \delta + A(\dot{\psi} + \dot{\phi} \cos \delta)\dot{\phi} \sin \delta - m\ddot{s}\, \ell \sin \delta = 0, \qquad (14)$$

$$I\dot{\phi}\sin^2\delta + A(\dot{\psi} + \dot{\phi}\cos\delta)\cos\delta = C_1, \tag{15}$$

$$\dot{\psi} + \dot{\phi}\cos\delta = C_2, \tag{16}$$

where C_1 and C_2 are integration constants. Equations (14) and (15) are identical to those of Thomas, while for (16) Thomas gives

$$\dot{\psi} + \dot{\phi} = \dot{\alpha}, \tag{17}$$

where $\alpha = \pi s/nr$; here n is the twist of the rifling in calibers/turn. Since $\cos\delta$ is, for all practical purposes, equal to unity, the difference between the left-hand sides of Equations (16) and (17) is not important. However, the right-hand side of (17) is not a constant. As a consequence, (17) is not satisfactory under the assumptions made by Thomas.

Under all conditions of the motion of the projectile (where the lands and rotating band are intact), the torque on the rotating band due to forces on the band from the groove walls (see Figure 2 where these forces are indicated by X_i^*) acts at all times to constrain the value of ψ so that

$$\dot{\psi} = \frac{\pi\dot{s}}{rn} \equiv \Omega(s), \tag{18}$$

where $\dot{s}$ is the velocity of the point C (at the center of the rotating band on the projectile axis) along the gun tube axis. Except for the initial deformation of the rotating band, the band does not constrain $\dot{\phi}$.

Under the conditions of no impact or reaction forces, therefore, we obtain, substituting for (18) in Equations (11), (12), and (13) and setting all the impact and reaction forces to zero,

$$I\ddot{\delta} - I\dot{\phi}^2\sin\delta\cos\delta + A(\Omega + \dot{\phi}\cos\delta)\dot{\phi}\sin\delta - m\ddot{s}\,\ell\sin\delta = 0, \tag{19}$$

$$I\dot{\phi}\sin^2\delta + A(\Omega + \dot{\phi}\cos\delta)\cos\delta = \cos\delta\int T dt, \tag{20}$$

$$A(\Omega + \dot{\phi}\cos\delta) = \int T dt. \tag{21}$$

If we eliminate the integral over T from Equations (20 and 21), we obtain, repeating from above to give the full set of equations,

$$(I\ddot{\delta} - I\dot{\phi}^2\sin\delta\cos\delta - m\ddot{s}\ell\sin\delta)\dot{\delta} + \dot{\phi}\frac{d}{dt}(I\dot{\phi}\sin^2\delta) = 0, \tag{22}$$

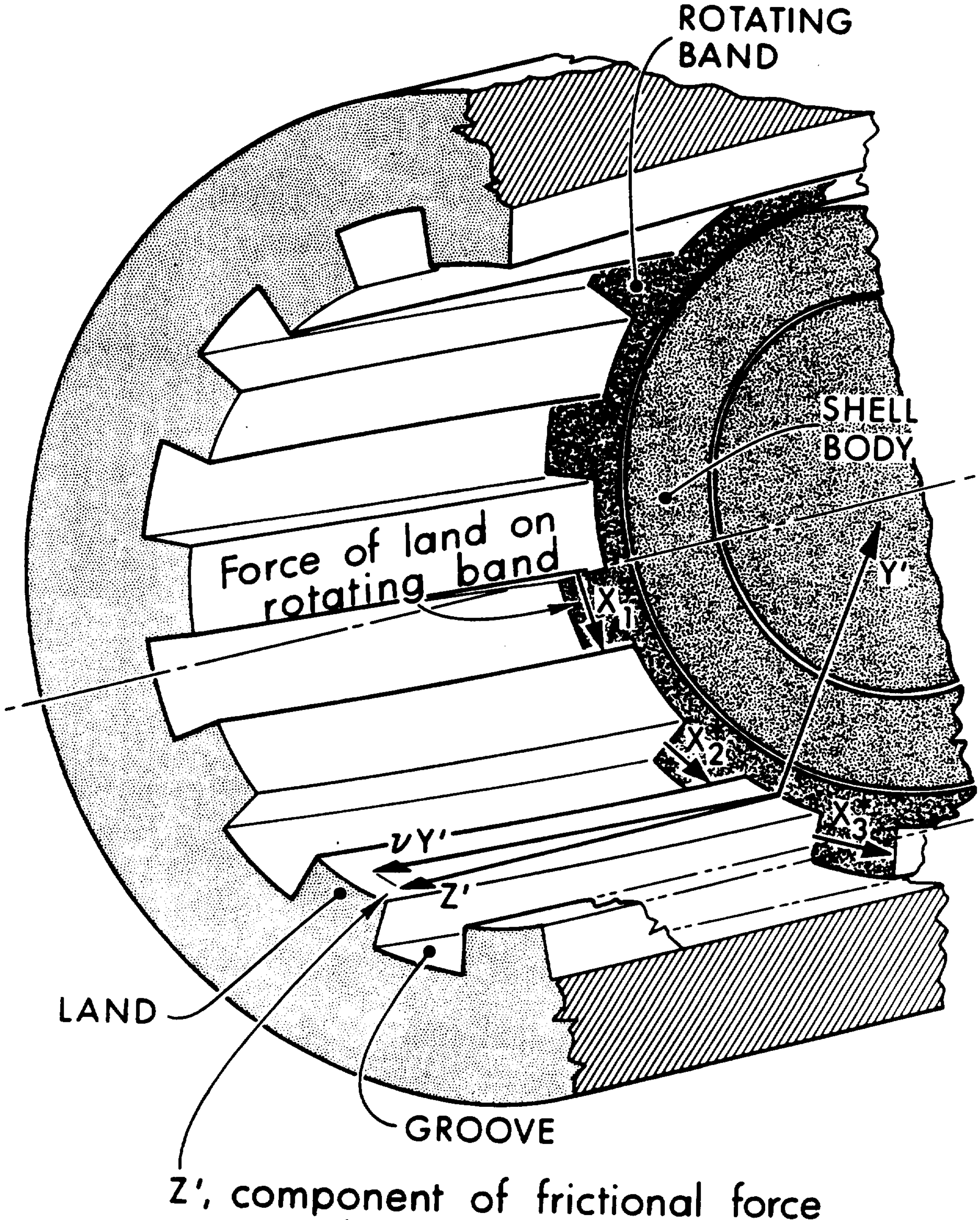

Figure 2. Sketch of the region of contact between the gun tube bore and the rotating band of a shell. The forces X_1^*, X_2^*, X_3^*... acting through the moment arm r give rise to the torque T': $T' = r\,\Sigma_i X_i^*$. Balloting impacts give rise to a force Y' at the rotating band, friction results in the force $\nu Y'$ having a component in the direction of travel of the shell (parallel to the bore axis), Z'.

$$\frac{d}{dt}\,(I\dot{\phi}\,\sin^2\delta)\; - \;A(\Omega + \dot{\phi}\,\cos\,\delta)\;\sin\,\delta\;\dot{\delta} = 0, \tag{23}$$

$$\Omega = \pi\dot{s}/rn, \tag{24}$$

and

$$T = \frac{d}{dt}\,[A(\Omega + \dot{\phi}\,\cos\,\delta)]. \tag{25}$$

This last equation can be used to determine the forces tending to shear the lands along planes parallel to the top of the lands.

Equations (22) and (23) can be put in an interesting and simple form, although one that is not solvable in closed form,

$$\ddot{\delta}/\sin\,\delta = S - \Omega'\dot{\phi} - I'\dot{\phi}^2\,\cos\,\delta, \tag{26}$$

where

$$I' = \frac{A}{I} - 1, \tag{27}$$

$$\Omega' = A\Omega/I, \tag{28}$$

and

$$S \;= m\ddot{s}\ell/I. \tag{29}$$

If we multiply Equation (26) through by I, the $\dot{\phi}^2$ term will be of the order of (but not equal to) the kinetic energy in the precession motion, the term in Ω' will be of the order of the spin kinetic energy reduced by a factor ϕ/Ω and S will be the increase in the kinetic energy of the projectile in traveling a distance ℓ. Now the kinetic energy E_ψ in the spin of the projectile in terms of k_a, the radius of gyration about the shell axis and the translational kinetic energy E_s, is

$$E_\psi = \frac{1}{2}\,A\Omega^2 = E_s(k_a\pi/rn)^2. \tag{30}$$

Thus, in terms of an effective length of the gun L_e as would be required to achieve the shell translational energy E_s at a constant force of $m\ddot{s}$,

$$\frac{\Omega'\dot{\phi}}{S} = \frac{2L_e\dot{\phi}}{\ell\Omega}\left(\frac{k_a\pi}{rn}\right)^2. \tag{31}$$

Since for much of the projectile travel n^2 is much larger than L_e/ℓ, the term S dominates the right side of Equation (26). Assuming S is

constant during the swing of projectile from the axis to the gun tube wall and $\sin \delta \approx \delta$, we obtain the approximate expression

$$\ddot{\delta} = S \sin \delta \tag{32}$$

having a solution for $\dot{\delta} = \dot{\delta}_o$ and $\delta = 0$ at $t = 0$,

$$\delta = \frac{\dot{\delta}_o}{2\sqrt{S}} \left(e^{\sqrt{S}t} - e^{-\sqrt{S}t} \right). \tag{33}$$

With this function determined and substituted into Equation (26), it is possible to obtain an approximate solution for ϕ. However, such solutions are of limited value in the practical case in which the principal characteristics of the motion are due to the impacts and the reaction forces acting on the rotating band.

Let us now turn to the expressions for the forces and torques acting on the shell in the general case which includes impacts and reaction forces. The Hertz[5] theory of impact can be employed advantageously to obtain a good expression for Y acting during impact together with Raman's[6] formulas for the coefficient of restitution. Gay's expression for Y is

$$Y = \begin{cases} \left.\begin{matrix} -K_c(\delta - \Delta), & \dot{\delta} > 0 \\ -K_r(\delta - \Delta), & \dot{\delta} < 0 \end{matrix}\right\} & \delta > \Delta \\ 0, \quad \delta \leq \Delta & \end{cases} \tag{34}$$

for which the coefficient of restitution e can be expressed as

$$e = (K_r/K_c)^{1/2}. \tag{35}$$

The X and Z forces are dependent on the coefficient of friction μ and on the direction of motion* of the point on the bourrelet in contact with the gun tube. We have, after Thomas (Reference 2)

$$X^2 + Z^2 = \mu^2 Y^2 \tag{36}$$

**that is, the sign of* $\dot{\delta}$.

and

$$\frac{X}{b(\Omega + \dot{\phi} \cos \delta) + a\dot{\phi} \sin \delta} = \frac{Z}{\dot{s}} \tag{37}$$

or as an approximation,

$$X \approx Zb\Omega/\dot{s}. \tag{38}$$

The force component Y' is given by the requirement that the sum of the forces acting in the y direction must be zero. However, since the shell is elastic (i.e., not infinitely rigid), there will be a time lag τ_e between the application of the forces Y and Y' to the shell. Thus we have that

$$Y'(t) = -Y(t - \tau_e) \tag{39}$$

where we can evaluate τ_e approximately, in terms of the shear wave velocity c_s in the shell, using the expression

$$\tau_e = \sqrt{\pi^2 b^2 + a^2}/c_s. \tag{40}$$

The frictional reaction forces X' and Z' are given in terms of the coefficient of friction ν between the rotating band and the tube wall by

$$X'^2 + Z'^2 = \nu^2 Y'^2 \tag{41}$$

and the relation for the ratio of X' to Z' in terms of velocity components

$$X' = Z' r\Omega/\dot{s} \tag{42}$$

which, on substitution into Equation (41), yields

$$Z' = \nu Y'/\sqrt{1 + (r\Omega/\dot{s})^2}. \tag{43}$$

The total torque $T' = Q_3$ acting in the z direction [i.e., the right-hand side of Equation (13)] is constrained by the requirement that

$$\dot{\psi} = \Omega = \pi\dot{s}/rn \tag{44}$$

as previously given. Thus we have that

$$T' = T + rX' - bX = \frac{d}{dt}\,[A(\Omega + \dot{\phi}\cos\delta)] \tag{45}$$

which, as mentioned above, can be evaluated in terms of the solution to the general problem to determine the shears acting across the lands.

The Equations (11-13), (34-37), and (39-45) provide the complete set of equations for computing the balloting of the shell where the function s(t) is given. A calculation of this motion will be presented in a separate paper. The following treatment is carried out to show the importance of these reaction forces.

III. GROWTH OF ENERGY IN THE TRANSVERSE MODE

The motion of the projectile in the bore of the gun that can lead to severe impact by the bourrelet against the lands is the motion along the δ coordinate. We will now develop the equations describing the growth in energy in this transverse motion about the point C (Figure 1) by first calculating the effects due to bourrelet impact against the lands and the subsequent reaction torques applied at the rotating band and then express this in the form of a time-averaged equation for this energy.

If during the time $t = 0$ to $t = t_1$ the shell moves $\delta = \Delta$ (i.e., bourrelet just making contact with the lands) to $\delta = \delta_{max}$ where $\dot{\delta} = 0$ as a result of the impact and if t_2 is the total duration of the contact between the bourrelet and the lands for the impact, we can write, integrating over Equation (11) for $t = 0$ to Δt, the time between impacts of the bourrelet with the lands

$$\int_0^{\Delta t} [I\ddot{\delta} - I\dot{\phi}^2 \sin\delta\cos\delta + A(\Omega + \dot{\phi}\cos\delta)\dot{\phi}\sin\delta - m\ddot{s}\,\ell\sin\delta]dt$$
$$= \int_0^{t_2} [(a\cos\delta - b\sin\delta)Y - (a\sin\delta + b\cos\delta)Z]dt + \int_{\tau_e}^{\tau_e + t_2} rZ'dt \tag{46}$$

where Z' is zero during the interval $\tau_e < t > \tau_e + t_2$ and Y and Z are zero for $t_2 < t < \Delta t$,

Let us first show that the second and third terms on the left side of Equation (46) do not contribute significantly to the motion computed during the impacts. We have from Equations (13) and (18)

$$\frac{d}{dt}\,[A(\Omega + \dot{\phi}\cos\delta] = -\,bX + rX' + T \tag{47}$$

where from Equations (18), (36), (38), (42), and (43)

$$X = \mu Y / \sqrt{1 + \frac{r^2 n^2}{b^2 \pi^2}} \tag{48}$$

and

$$X' = \nu Y' / \sqrt{1 + n^2/\pi^2}. \tag{49}$$

Since

$$b \approx r \tag{50}$$

we can write, using Equation (39) to obtain a result holding for a single cycle of the motion (that is to say, the action and reaction impulses)

$$\dot{\phi}\,\frac{d}{d\phi}\,[A(\Omega + \dot{\phi}\cos\delta)] \approx r\,\frac{\mu Y(t) - \nu Y(t - \tau_e)}{\sqrt{1 + n^2/\pi^2}} + T. \tag{51}$$

Noting that the change in δ is extremely small during each impact we have, integrating (51) twice,

$$\int_{o}^{t_2} [A(\Omega + \dot{\phi}\cos\delta)\ \dot{\phi}\sin\delta]dt + \int_{\tau_e}^{\tau_e + t_2} [A(\Omega + \dot{\phi}\cos\delta)\ \dot{\phi}\sin\delta]dt$$

$$\approx \frac{\mu\Delta - \nu\delta'}{\sqrt{1 + n^2/\pi^2}}\ r\,\overline{Y}\,(\Delta\phi)t_2 + (\Delta + \delta')\ T\ \Delta\phi \cdot t_2 \tag{52}$$

where δ' is the value of δ at $t = \tau_e + t_1$, $\Delta\phi$ is the equivalent angle that the shell moves through during the interval t_2 and $\overline{Y}$ is the average force applied in the y direction. Now subtracting Equation (13) from (12) gives

$$\frac{d}{dt}\left[I\,\dot{\phi}\,\sin^2\delta + A(\Omega + \dot{\phi}\cos\delta)\,(\cos\delta - 1)\right]$$

$$= (b - b\cos\delta - a\sin\delta)\,X + (\cos\delta - 1)\,r\,X' + (\cos\delta - 1)\,T. \tag{53}$$

Repeating the above procedure gives

$$\int_0^{t_2} [I\,\dot{\phi}^2 \sin\delta\cos\delta + A(\Omega + \dot{\phi}\cos\delta)\,\dot{\phi}\sin\delta]\,dt$$

$$+ \int_{\tau_e}^{\tau_e + t_2} [I\,\dot{\phi}^2 \sin\delta\cos\delta + A(\Omega + \dot{\phi}\cos\delta)\,\dot{\phi}\sin\delta]\,dt$$

$$\approx [(b\,\Delta - a)\,\mu + r\,\delta'\,\nu]\,\frac{\overline{Y}\,\Delta\phi \cdot t_2}{\sqrt{1 + n^2/\pi^2}} + (\Delta + \delta')\,T\,\,t_2\,\Delta\phi\,. \tag{54}$$

Thus from (52) and (54) we have

$$\int_0^{t_2} [-\,I\,\dot{\phi}\sin\delta\cos\delta + A(\Omega + \dot{\phi}\cos\delta)\,\dot{\phi}\sin\delta]\,dt$$

$$+ \int_{\tau_e}^{\tau_e + t_2} [-\,I\,\dot{\phi}^2 \sin\delta\cos\delta + A(\Omega + \dot{\phi}\cos\delta)\,\dot{\phi}\sin\delta]\,dt$$

$$\approx \left[\frac{(\mu\Delta - \nu\delta' + \mu a)}{\sqrt{1 + n^2/\pi^2}}\,\overline{Y} + (\Delta + \delta')\,T\right]\,t_2\,\Delta\phi$$

$$\approx \frac{\mu a\,\overline{Y}\,t_2\,\Delta\phi}{\sqrt{1 + n^2/\pi^2}}\,. \tag{55}$$

We will find below that this contribution is smaller than other terms by a factor $\Delta\phi / \sqrt{1 + n^2/\pi^2}$ where $\Delta\phi$ is the angle through which the shell precesses during the impact of the bourrelet.

Returning again to Equation (46) we see that for complete cycles of the motion for which δ varies between $\pm\ \delta_{max}$, the second, third and fourth terms contribute no time averaged change to the motion (although these terms do effect the value of Δt) since $\sin \delta$ changes sign for negative values of δ. This cannot be stated if δ always satisfies

$$\delta > 0 \ , \tag{56}$$

in which a more detailed calculation using numerical techniques is required. The condition in (56) will occur if no significant initial balloting occurs. Under such conditions it is expected that bore riding will be a likely mode of projectile motion in the bore.

Making use of Equations (18), (36), (38) and (44) and setting

$$a \cos \delta - b \sin \delta \approx a \tag{57}$$

$$a \sin \delta + b \cos \delta \approx b \tag{58}$$

and

$$r \approx b \ . \tag{59}$$

Equation (46) in the time interval $0 \leq t \leq t_2$ becomes

$$\left[a - b\ \mu / \sqrt{1 + \pi^2/n^2} \right] \int_0^{t_2} Y\, dt = I \int_0^{t_2} \ddot{\delta}\, dt \ . \tag{60}$$

Now the coefficient of restitution may be more generally defined than as given in Equation (35). In general we have

$$\int_0^{t_1} Y\, dt = \left(2I \int_\Delta^{\delta_{max}} Y\, d\delta \right)^{1/2} = \left(2I \int_\Delta^{\delta_{max}} f_1(\delta)\, d\delta \right)^{1/2} \tag{61}$$

while

$$\int_{t_1}^{t_2} Y\, dt = \left(2I \int_{\delta_{max}}^{\Delta} f_2(\delta)\, d\delta \right)^{1/2} = e \left(2I \int_\Delta^{\delta_{max}} f_1(\delta)\, d\delta \right)^{1/2} . \tag{62}$$

As a result, Equation (61) multiplied through by the moment arm $[a - b\,\mu/\sqrt{1 + \pi^2/n^2}]$ gives the loss in angular momentum as the shell momentarily stops while Equation (62) multiplied through by this moment arm gives an expression for the recovered angular momentum. Thus Equation (60) gives us

$$\left[a - b\,\mu/\sqrt{1 + \pi^2/n^2}\right]\int_o^{t_2} Y\,dt = -\,(1 - e)\,I\,\dot{\delta}_i \tag{63}$$

where $\dot{\delta}_i$ is the angular velocity at impact. This can also be expressed in terms of an average torque $\overline{T}_y$ acting for the time interval Δt between impacts:

$$\overline{T}_y\,\Delta t = (e - 1)\,I\,\dot{\delta}_i\ . \tag{64}$$

The last term in Equation (46) is the integral over the torque rZ'. If

$$t_2 \le \tau_e \le \Delta t \tag{65}$$

then both the direction of the torque and the displacement of the shell in the δ direction will retain their signs unchanged during the interval $t = \tau_e$ to $\tau_e + t_2$. It was the change in directions of motion that gave rise to the sign change resulting in the factor (1-e) in Equation (63). The rZ' torque accelerates the angular motion of the shell in the δ direction throughout the time interval τ_e to $\tau_e + t_2$. During this time interval Y and Z will be zero. We have, therefore,

$$r\int_{\tau_e}^{\tau_e+t_2} Z'\,dt = I\int_{\tau_e}^{\tau_e+t_2} \ddot{\delta}\,dt\ . \tag{66}$$

From Equations (39), (43) and (44) we have

$$Z'(t) = -\,\nu\,Y(t - \tau_e)/\sqrt{1 + \pi^2/n^2}\ . \tag{67}$$

Thus, the time average torque $\overline{T}_z$ will be

$$\overline{T}_z \; \Delta t = r \int_{\tau_e}^{\tau_e + t_2} Z' \, dt = - \frac{\nu r}{\sqrt{1 + \pi^2/n^2}} \int_0^{t_2} Y \, dt$$

$$= \frac{(1+e) \; \nu \; r \; I \; \dot{\delta}_i}{a \sqrt{1 + \pi^2/n^2} - \mu b} = I \int_{\tau_e}^{\tau_e + t_2} \ddot{\delta} \; dt. \tag{68}$$

Now let us introduce a quantity Θ defined by

$$\Theta(t) = \int_0^t |\dot{\delta}| \; dt \tag{69}$$

which is simply the total angle swept out by the shell in the δ direction, ignoring the direction of motion. Introducing this angle allows us to treat the impacts between the bourrelet and the bore in a time averaged way. From Equations (46), (64), (68) and (69) we have

$$I \; \ddot{\Theta} = \left[\frac{(1+e) \; \nu \; r}{a \sqrt{1 + \pi^2/n^2} - \mu b} + (e-1) \right] \frac{I \dot{\Theta}}{\Delta t} \; . \tag{70}$$

An expression for Δt can be obtained using Equation (33). Expanding Equation (33) in a series gives

$$\delta = \frac{\dot{\delta}_o}{2\sqrt{S}} \left(2 \sqrt{S} \; t + \cdots \right) \tag{71}$$

where $\dot{\delta}_o$ is the angular velocity at $\delta = 0$. If $\frac{1}{2} S \; \Delta t << 1$ we can write for a swing through an angel 2Δ:

$$\Delta t = 2\Delta / \dot{\delta}_o \; . \tag{72}$$

If we let c be the clearance on each side of the bourrelet then

$$\Delta = c/a \tag{73}$$

and

$$\Delta t = 2c/a\dot{\Theta} \ . \tag{74}$$

Substituting Equation (74) into (70) and writing

$$\varepsilon = \frac{1}{2} I \ \dot{\Theta}^2 \tag{75}$$

gives, if $t_2 < \tau_e < \Delta t$,

$$\frac{d\varepsilon}{d\Theta} = \gamma\varepsilon \tag{76}$$

where

$$\gamma = \frac{a}{c}\left[\frac{(1+e)\ \nu\ r}{a\sqrt{1 + \pi^2/n^2} - \mu b} + (e-1)\right] . \tag{77}$$

For an initial energy of $\varepsilon = \varepsilon_o$ at $\Theta = o$, $t = o$, integration of Equation (76) gives

$$\varepsilon = \varepsilon_o \ e^{\gamma\Theta} \ . \tag{78}$$

This can also be integrated a second time to yield

$$t = \frac{1}{\gamma}\sqrt{\frac{2I}{\varepsilon_o}}\left(1 - e^{-\gamma\Theta/2}\right) = \frac{1}{\gamma}\sqrt{\frac{2I}{\varepsilon_o}}\left(1 - \sqrt{\varepsilon_o/\varepsilon}\right) \tag{79}$$

or, more conveniently,

$$\varepsilon = \varepsilon_o/(1 - \sqrt{\varepsilon_o/2I}\ \gamma t)^2 \ . \tag{80}$$

This growth in the energy in the transverse mode requires that several conditions be met:

$$\text{(i)} \quad \gamma > 0 \tag{81}$$

$$\text{(ii)} \quad \tau_e > t_2 \tag{82}$$

$$\text{(iii)} \quad \tau_e < 2c/a\dot{\theta} \text{ for all } \dot{\theta} \tag{83}$$

$$\text{(iv)} \quad \varepsilon_o > 0 \; . \tag{84}$$

Condition (iv) implies that some additional mechanism must initiate the balloting motion. Clearly balloting, particularly the development of violent balloting, is not to be expected in all cases. However, once this motion is initiated, the energy of the system rapidly moves into this mode. Figure 3 shows how this energy increases with motion of the projectile down the gun tube until some damage to the shell or to the bore removes the energy. Subsequently, the energy in this motion again increases until the shell exits or general failure occurs.

IV. INITIAL MOTION OF THE PROJECTILE

To calculate the growth of energy in the transverse mode using Equation (80), it is necessary to calculate the initial energy ε_o in that mode. That energy comes from the initial swing of the shell. Since initially $\dot{\phi}$, Y, Z and Z' are all zero, Equation (11) gives

$$I \ddot{\delta} = \frac{d\varepsilon}{d\delta} = m \ddot{s} \ell \sin (\delta + \delta_c) \tag{85}$$

where δ_c designates the angle between the shell axis and the center of gravity projected on the y-z plane. For cases in which the $\ddot{s}$ can be treated as constant we have, taking $\varepsilon = o$, $\delta = o$ at $t = o$:

$$\begin{aligned} \varepsilon_o &= m \ddot{s} \ell (1 - \cos \Delta) \\ &\approx m \ddot{s} \ell c^2/2 a^2 . \end{aligned} \tag{86}$$

In general, this is not the case however. It is necessary to take account of the change in pressure as a function of time, requiring numerical methods.

It will be noticed that unless the initial value of the center of gravity yaw, $\delta_o + \delta_c$, differs from zero, there will be no motion in the δ direction.

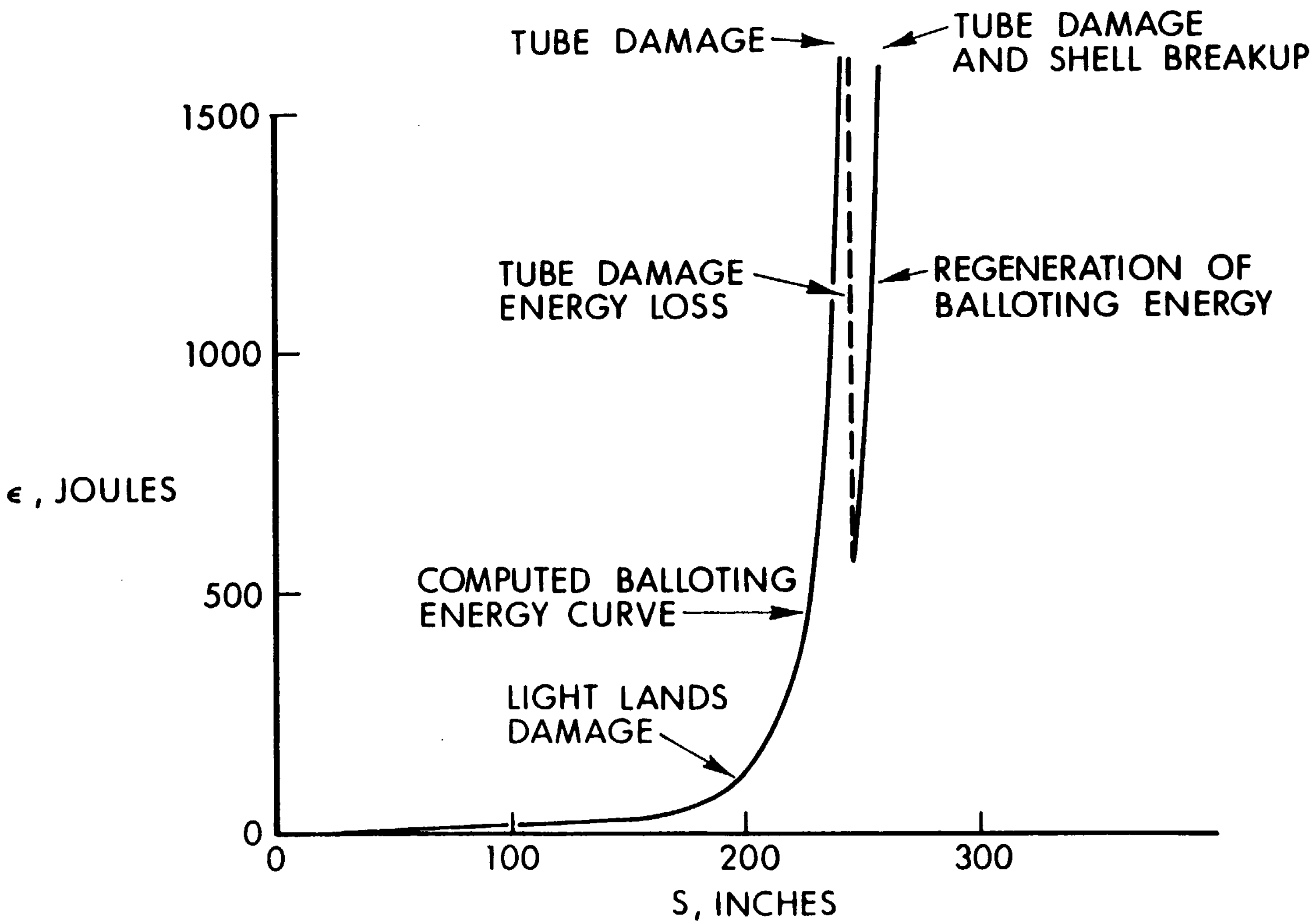

Figure 3. Calculation of balloting energy ε as a function of shell position s in the 8" Howitzer tube for a zone 9 charge as computed using Eq. (80) and the results of the integration of Eq. (85). Values of constants used are as given in the text with $I = 5.245$ slug ft^2 and $\gamma = 140.6$. The energy rapidly rises probably producing the first bands damage (see Para. a, Section VI) without significant loss in energy. The major damage (see Para b, Section VI) would remove a significant amount of the balloting energy as indicated by the dashed line. Regeneration of balloting would lead to further tube damage (see Para c, Section VI).

The closer $\delta_o + \delta_c$ is to zero initially the longer will be the time until the shell bourrelet strikes the bore wall. This delay allows the pressure and thus $\ddot{s}$ to increase. The likelyhood that δ_o will be sufficiently small to allow for a sufficient delay in this impact is small and largely determines the probability that ε_o will be sufficiently large to cause significant balloting.

Figure 4 shows the results of numerical integration of Equation (85) for the conditions of an 8" M106 HE shell launched with a zone 9 charge producing a pressure history as given in Figure 5. The shell parameters as used here were:

$$\text{weight } mg = 201.\ \text{lbs}$$
$$I = 5.245 \text{ slug ft}^2$$
$$c = 0.0142''$$
$$a = 9.08''$$
$$\ell = 5.678''.$$

In addition, Figure 4 together with the results in Figure 6 gives the probability that the initial value of $\delta = \delta_o$ will lead to gun damage assuming all possible initial positions are equally likely. The probability P is given by the ratio of solid angles giving rise to sufficient energy to cause damage Ω_d divided by the total solid angle allowed for the initial shell position:

$$P = \Omega_d/\pi\Delta^2 \ . \tag{87}$$

Since t in Equation (80) will be a function of δ_o one must compute ε and evaluate Equation (87). This result is shown in Figure 6.

V. CALCULATION OF THE COEFFICIENT OF RESTITUTION

An examination of Equations (77) and (81) shows that the coefficient of restitution is an important parameter of balloting motion. Accurate calculations of this motion properly requires an experimental determination of this parameter for the particular conditions of the shell in the bore. An approximation to the value can be obtained, nevertheless by adopting the formulas for the coefficient of restitution developed by Raman[6] which were based on Hertz's[5] (see also the treatise on elasticity by Love[7]) theory of impact.

From Hertz's Theory we have for the duration of an impact between two spheres of radii r_1 and r_2:

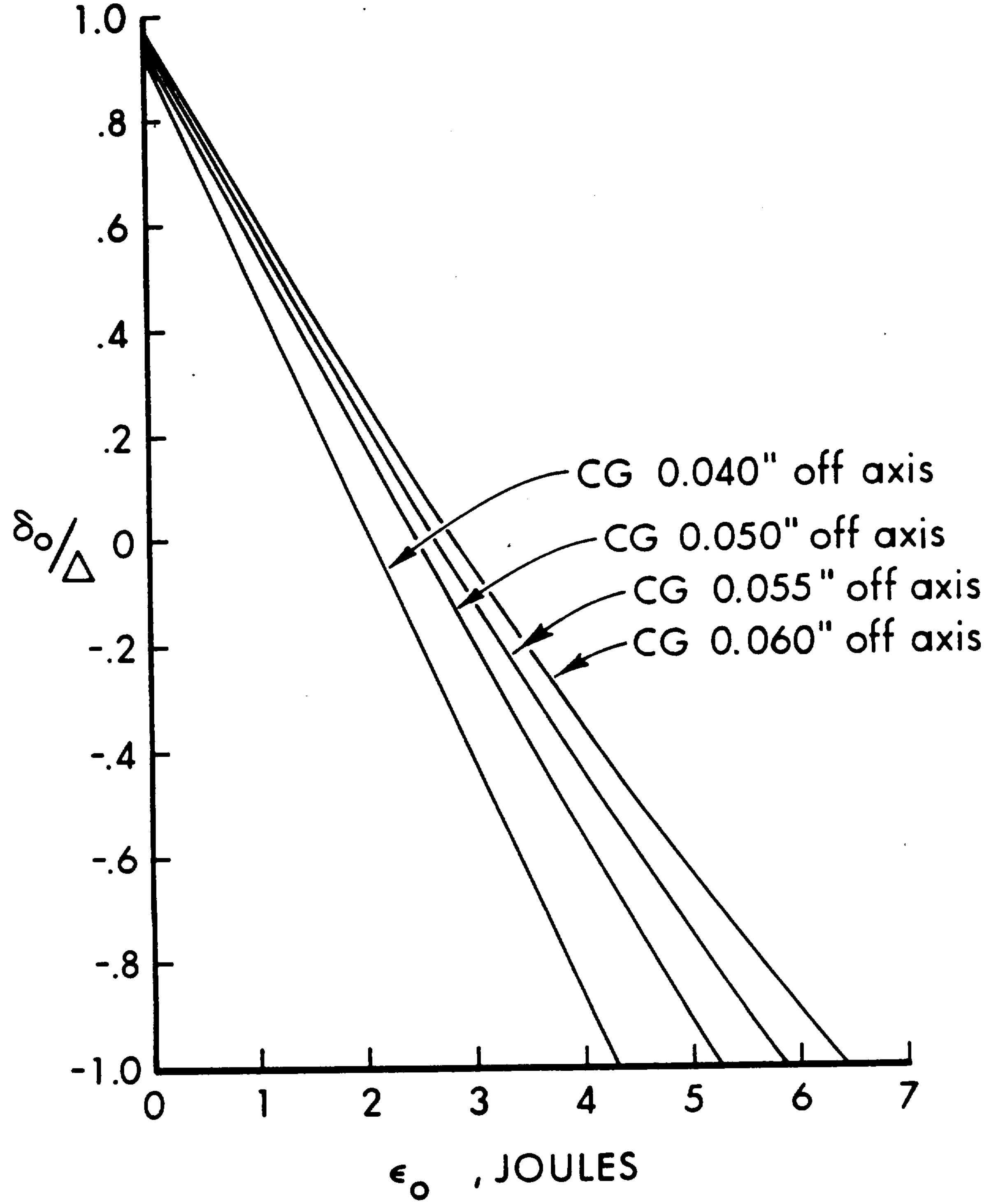

Figure 4. Plot of the energy ε_o in the balloting mode at first impact with the gun bore as a function initial shell altitude (yaw angle δ_o divided by the maximum yaw angle Δ) as computed from Eq. (85) for the 8" M106 HE shell launched with a zero 9 charge in the XM201 Howitzer tube. The pressure history used for these calculations is given in Figure 5.

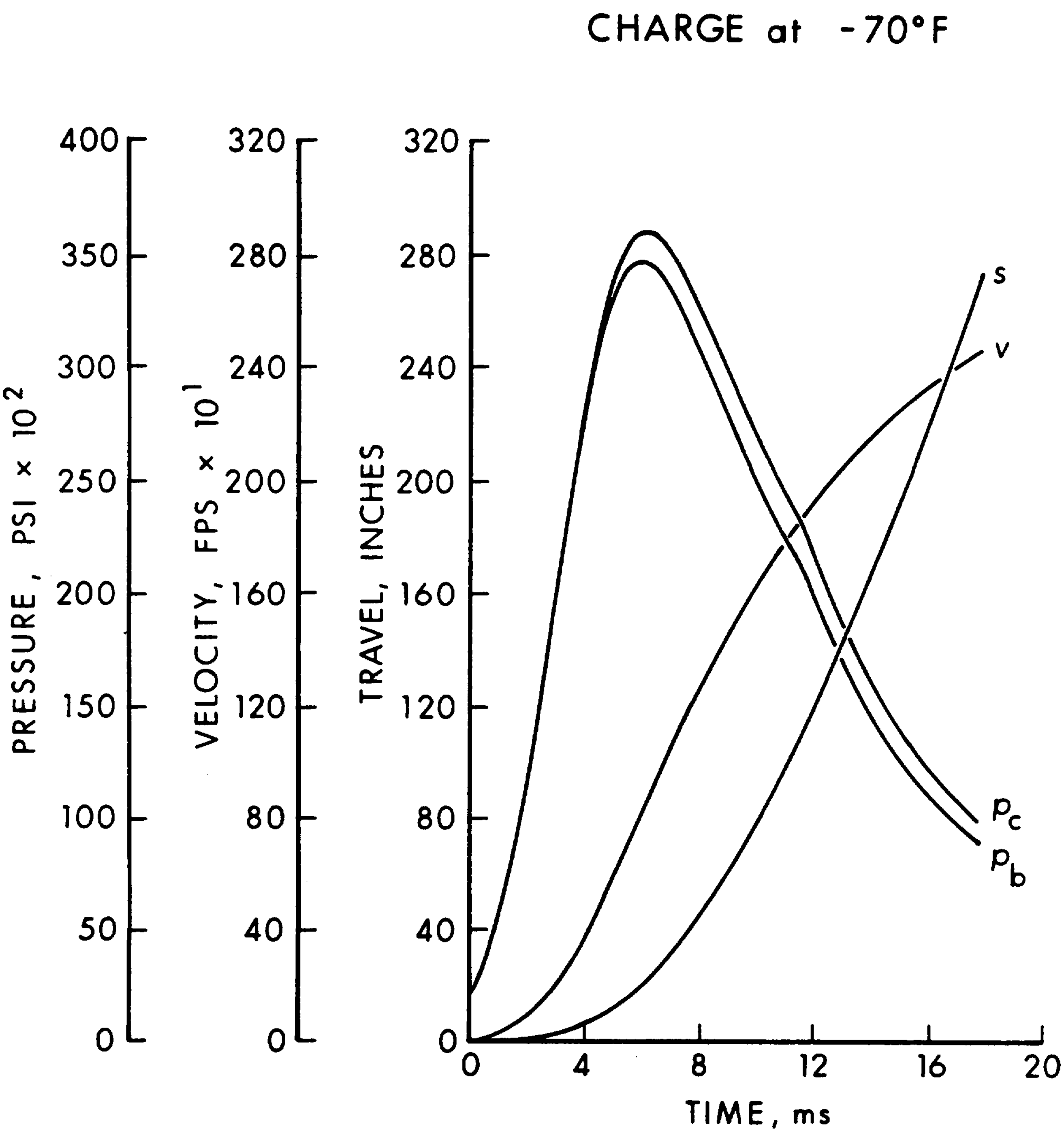

Figure 5. Plot of the pressure time curves computed by L.D. Heppner of MTD for the 8" M106 HE shell launched with an XM188 zone 9 charge at -70°F in the XM201 Howitzer tube. The chamber pressure p_c, base pressure p_b, velocity v and position z of the shell in the tube are shown. These curves were used to compute the results given in Figures 4 and 6.

$$t_2 = 2.94 \left[\frac{15}{16} \left(\frac{1 - \sigma_1^2}{q_1} + \frac{1 - \sigma_2^2}{q_2} \right) \frac{m_1 \, m_2}{m_1 + m_2} \right]^{2/5} \left[\frac{1}{v} \left(\frac{r_1 + r_2}{r_1 \, r_2} \right) \right]^{1/5} \qquad (88)$$

where

σ_1, σ_2 = The Poisson's ratio for the first and second ball, respectively.

q_1, q_2 = The Young's modulus for the first and second ball, respectively.

m_1, m_2 = mass of the first and second impacting ball, respectively.

v = velocity of impact.

Raman modified Equation (88) to apply to the case of a ball of mass m_2 impacting against a plate of thickness $2b_1$ and density ρ_1 obtaining

$$e = \frac{b_1 \, \rho_1 \, a^2 - \kappa \, m_2}{b_1 \, \rho_1 \, a^2 + \kappa \, m_2} \qquad (89)$$

where κ is a coefficient describing the dominant vibrational mode, a^2 is

$$a^2 = \pi \, t_2 \, b_1 \sqrt{q_1 / 3 \, \rho_1 \, (1 - \sigma_1^2)} \qquad (90)$$

and where Raman has taken the approximations

$$m = m_2 \approx \frac{m_1 \, m_2}{m_1 + m_2} \qquad (91)$$

and

$$r_2 \approx \frac{r_1 \, r_2}{r_1 + r_2} \qquad (92)$$

Raman gives values of κ = 0.56, 0.44 and 0.39.

Because of the special geometry with which we are dealing in the case of a shell bourrelet impacting against the gun tube wall, we will also assume Equation (91) holds and replace the term $r_1 \, r_2/(r_1 + r_2)$ by

$$\xi \sqrt{\ell_b \left(\frac{r_1 \, r_2}{r_1 - r_2} \right)} \qquad (93)$$

where ξ is the portion of the gun tube lands to lands plus grooves area and ℓ_b is the radius of curvature of the bourrelet contacting the lands. The expression in Equation (93) provides a correction for the area contacting the lands on impact which corrects for the "negative" curvature of the bore surface (compared to that used as a basis in the Hertz calculations); since the radii of curvature will differ in the two direction of the two major axes, this must be reflected by introducing ℓ_b to scale the radius of curvature in the direction parallel to the tube axis. We thus obtain:

$$t_2 = 2.94 \left[\frac{15}{16} \left(\frac{1 - \sigma_2^2}{q_1} + \frac{1 - \sigma_2^2}{q_2} \right) m \right]^{2/5} \left[\frac{1}{\xi v} \sqrt{\frac{(r - b)}{r\, b\, \ell_b}} \right]^{1/5} \tag{94}$$

which together with Equations (89) and (90) provide an expression for the coefficient of restitution.

In the case of the 8" Howitzer XM201 with the M106 HE shell we have

$$\rho_1 = 7.85 \text{ g/cm}^3 = 0.2833 \text{ lbs/in}^3$$

$$\sigma_1 = \sigma_2 = 0.3$$

$$q_1 = q_2 = 3 \times 10^7 \text{ psi}$$

$$\xi = 0.4$$

$$r = 4.0015''$$

$$b = 3.9875''$$

$$\ell_b = 64.0''$$

$$b_1 = 2.252'' \text{ (near the breech of the gun)}$$

and from Figure 4 an average value for ε_o is 3 Joules giving $v = 8.34''$/sec. These values gives:

$$t_2 = 7.35 \times 10^{-4} \text{ sec} \tag{95}$$

and

$$e = \begin{cases} 0.566 \ , & \kappa = 0.56 \\ 0.642 \ , & \kappa = 0.44 \\ 0.676 \ , & \kappa = 0.39. \end{cases} \tag{96}$$

In the calculations below we have used $e = 0.7$.

VI. APPLICATION TO THE 8" HOWITZER TUBE XM201 MECHANICAL FAILURE OF APRIL 11, 1974

On April 11, 1974, while firing with temperature conditioned M106 HE the shell broke up in the gun tube and produced moderate to heavy damage to the tube. The test of the Equipment Performance Report is given in Appendix I.

Damage to the tube as described in the Star gage report of 11 April, 1974 (by P. Booth, J. McWilliams, and R. Kane, Appendix I) itemizes:

a. "Driving and non-driving edge of one land sheared away between 185.00" and 186.75" from (RFT)." This shear is symmetric to the center line of the land (midpoint of damage at 185.88".)

b. "Heavy to moderate damage to lands encircling bore between 249.75" and 258.50" from (RFT). Damages consist of lands flattened and compressed... From this point forward edges of lands have split away and sprung out from the base of the land for a distance of about 8"...." Again, shearing is in general symmetric about the land center line. Shear planes generally meet at the center line toward breach end of shear and taper off toward land edges toward the muzzle end of the shear (midpoint of damage at 254.12", 68.25" from midpoint of last damage).

c. "Moderate to light [similar to "b" above] damage to 8 lands between 273.75" and 280.75" (RFT)." (Midpoint of damage at 277.25"; this is 23.12" from midpoint of last damage).

d. The maximum increase in bore diameter was 20/1000th inch for the lands and 7/1000ths for the grooves.

The shear and flattening damage to the lands indicates a high loading normal to the top surface of the damaged lands. Since the magnitude of the increase in tube diameter was slight, initiation of the HE as a cause of damage is not indicated. Similar (principal) damage appearing in several parts of the tube also argues against HE initiation. The residual TNT after the shot and the fact that the fuze exited the muzzle intact also argue against HE initiation as a cause of the damage. The time-pressure record (see Appendix I) for this shot was, except for small increase late in the strain record, normal. The small spikes in the strain record are consistant with the present interpretation (this is discussed further in Appendix I).

The play in the shell between the bourrelet and lands is estimated to have been 0.014" (all around the shell; i.e., a full swing of the shell would have been 0.028") before impact engraving and about 0.024" to 0.03" after engraving of the bourrelet. These values are obtained from the tube lands diameter in the undamaged regions as given by the Stargage report to be 8.003", the average bourrelet diameter measured on other shells in lot CSK 1-136 yielding an average of 7.988" at normal

temperature and 7.975" at -70°F (the shell temperature at failure). Computer stress-strain calculations for the M106 shell show an increase in bourrelet diameter of +0.0046" at peak pressure and -0.0042" at zero base pressure. At the point of major damage, normal bourrelet deformation would be small (about 0.001").

The tube tested (XM201) extends beyond old tube (in M110 Howitzer). Damage of the type described in paragraph 2b,c above would lie beyond the 210.74" (RFT) position of the muzzle for the M110 Howitzer tube. Characteristic dimensions for shells from the same lot as that which broke up are listed in Table 1.

Using this data let us now determine if the conditions for the development of balloting motion after Equation (80) as given by Equations (81) - (84) were satisfied. As shown by the results in Figure 4, condition (Equation 84) is generally satisfied. The sound velocity in the shell for the calculation of τ_e using Equation (40) is given by (see Raman[6])

$$c_s = \frac{\pi\, b_2}{\lambda} \sqrt{q_2/3\, \rho_2\, (1 - \sigma_2^2)} \qquad (97)$$

where the subscripts refer to the appropriate values for the shell. The quantity b_2 is the wall thickness of the shell and λ is wave length of flexural waves in the shell. The wave length of the principal mode will be:

$$\lambda = 4 \sqrt{\pi^2\, b^2 + a^2}\ . \qquad (98)$$

Using Equations (97) and (98) in Equation (40) gives:

$$\tau_e = 1.64 \times 10^{-3} \text{ sec.} \qquad (99)$$

Comparing Equations (95) to (99) we see that condition (82) is satisfied. Further, using that same value of ε in Equation (75) we see that $2c/a\dot{\Theta} = 4.72 \times 10^{-3}$ sec so that condition (83) is satisfied. Finally substituting for γ, using handbook values of $\nu = 0.55$ and $\mu = 0.50$ and $n = 20$, we obtain

$$\gamma = 140.6 \qquad (100)$$

which, being positive, satisfies the condition of Equation (81).

It should be noted that the coefficients of friction are not constants under the conditions of gun firing. A precise calculation,

Table 1. 8" Shell M106 with Fuze M73 Dummy

Lot No. CSK-1-136 (10P 17-75) with 1 Suppl. Change

Shell No.	Diameters (inches) Front: Bourr.	Above Band	Below Band	Rotating Band	Band Lip	Total Weight (lbs)	Total Length (inches)	C of G (inches) From Base of Shell	Moments Of Inertia (lbs Ft^2): Axial	Transverse
136 V	7.985	7.972	7.973	8.141	8.281	200.48	35.00	12.17	12.7266	101.3459
H	7.985	7.972	7.973	8.141	8.281					
150 V	7.989	7.971	7.973	8.138	8.279	201.14	35.10	12.16	12.7266	101.7727
H	7.988	7.971	7.974	8.139	8.279					
151 V	7.988	7.971	7.973	8.139	8.279	201.28	35.03	12.19	12.7003	101.5592
H	7.988	7.972	7.973	8.139	8.279					

using a computer to follow the details of the balloting should make use of variations in μ and ν as may be obtained from Bowden and Taylor[8], for example.

Now using Equation (100) in Equation (80), taking ε_o from the computed values given in Figure 4 and allowing for the requirement that t as used in Equation (80) satisfy the constraint:

$$t_s = t + \Delta t_o \qquad (101)$$

where Δt_o is the time to develop the initial energy ε_o and t_s is the time at which the shell exists the muzzle we can compute the energy ε at the end of the muzzle as a function of δ_o. The result as computed is given in Figure 6. The value of δ_c in Equation (85) is taken as 0.0070, 0.0088, 0.0097 and 0.0106 corresponding to a CG off the geometric axis by 0.040", 0.050", 0.055" and 0.060". The results show the extreme sensitivity of the results to the location of the CG.

From Figure 6, we see that the balloting energy exceeds 10^4 Joules for a CG 0.055" off axis if the initial position of the shell axis lies in a region between δ_o = 0.0011 and 0.0016 radiams as shown in Figure 6. Such an energy corresponds to that required to produce the observed deformation of the gun tube (8 to 10 K Joules) as occurred in the XM201 8" Howitzer mechanical failure. Further, the pressure acting on the lands due to the impact of a shell with 2800J in the balloting mode on the 3.49 in^2 (projected area) contacting the lands at the bourrelet in a severe impact will subject the lands to a pressure of 150Ksi which is sufficient to cause the observed shear damage to the lands.

If all possible initial values of δ_o are equally likely, the probability that the shell will lie in this region is about 0.068 for a CG 0.055" off axis corresponding to a chance of 1 in 15 for the build up in energy by balloting. In addition, δ_c must be off axis as mentioned above. A value of 0.055" off axis for the CG of the shell is not probable but can occur under the allowed tolerance specified for the shell. The rough handling of the shell which led to break up of the charge in the shell also can contribute to the off axis CG. It should be noted that this value of 0.055" is approximate since the coefficient of friction can vary considerably from the values used.

We thus find that the equations obtained in this report allow one to calculate the conditions under which balloting can build to severe levels and produce gun tube damage.

The calculations indicate the following scenario for the XM201 8" Howitzer mechanical failure. Onset of balloting was initiated by an initial high torque due to CG lying off the shell axis. Balloting under the test conditions developed severely enough to cause early engraving of the bourrelet of the shell, leading to increased play at the bourrelet.

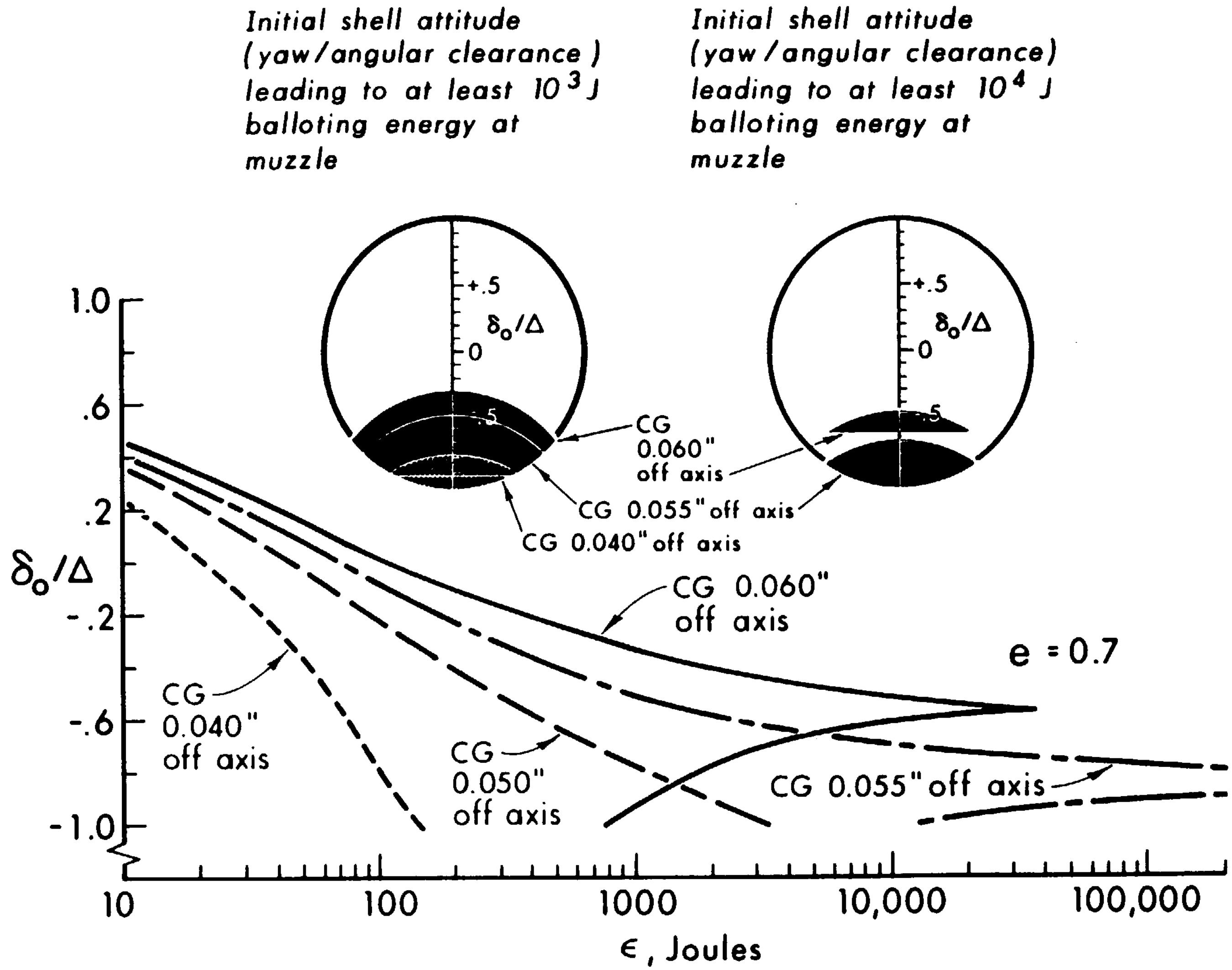

Figure 6. Plot of the energy ε in the balloting mode at the muzzle of the gun tube as a function of the initial shell altitude (initial yaw angle divided by the maximum possible yaw) computed from Eqs. (80) and (101) using the results the integration of Eq. (85) shown in Figure 4 and with a coefficient of restitution e of 0.7 and γ = 140.6 as appropriate for the conditions of the 8" M106 HE shell at -70°F, the XM188 zone 9 charge at -70°F fired in the XM201 Howitzer tube. Several values of position of the shell's center of gravity off axis are used in these calculations. The CG, shell axis and bore axis are all assumed to lie in a single plane. The corresponding solid angles leading to at least 10^3J and 10^4J for all possible initial positions (i.e., CG, shell axis and bore axis not necessarily in a plane) are shown in the inserts. It will be noticed that only a narrow range values of the CG off axis leads to large balloting energies and only if the shell as initially loaded lies in an allowed altitude.

The resulting enhanced balloting caused the shearing of the lands at 185" (RFT) noted in Appendix I and perhaps some shell fracture while momentarily reducing the balloting. Further increase in balloting resulted in the damage between 249.75" and 258.50" (RFT). The major fracturing of the shell would be expected at this point, but not shell breakup. The damage at 277" (RFT) indicates a final buildup of balloting, producing the typical shear failure of the lands and the final breakup of the shell.

As already mentioned, the coefficient of friction can rise significantly as balloting increases pressure between the shell and bore of the gun which can result in binding between the metal surfaces which causes striping of the metal. Such failure of the metal is observed 9" before the major damage and agin 9" before the final damage. The appearance of this damage is exactly the same as is shown in Figure 5, plate XXXII of Reference 8. Note that 9" is the distance between the rotating band and the bourrelet.

It is significant that the damage to the lands occurs in the portion of the XM201 tube that has been added to the old 8" Howitzer tube. This alone is not the cause of the failure, however. The large clearance of the shell and asymmetry caused by an off axis CG are additional causes of the growth of violent balloting that produced breakup of the shell and gun tube damage.

A six degree of freedom formulation for the transverse motion of an accelerating shell has been given by Chu and Soechting[8], and extended in an application to the 8 inch projectile in the XM201, M2A2 gun tube, MK-16 and MCLG Gun by Chu[9]. These papers have the advantage that the effect of shell flexure as a function of shell wall thickness and gun tube flexure are included in the formulation. The formulation does not include the coefficient of restitution nor the very important time delay between bourrelet impact and the reaction force at the rotating band. Excluding this latter effect removes the frictional forces as significant contributors to the development of balloting motion. Although these formulations are quite superior to those of Reno[1] and Thomas[2], no terms are included that would significantly (i.e., by an order of magnitude) alter their results for the lateral force exerted by the shell on the bore as computed by Gay[3] for the case of bore riding. Gay does not find the large forces reported by References 9 and 10. The balloting process involving the buildup of energy in the transverse mode through cumulative, driven impacts by a shell with (excessive) clearance at the bourrelet is not incorporated into the formulations of References 9 and 10. This should not detract from the significant accomplishment of Chu and Soechting's work.

ACKNOWLEDGMENTS

I wish to acknowledge the assistance and encouragement of Dr. Coy Glass who suggested this problem; Mr. Pak Yip who ran computer calculation and to Mr. Ronald Hendrickson and Mr. Robert Huddleston of MTD who furnished much of the data used in the calculations.

REFERENCES

1. F. V. Reno, "The Motion of the Axis of a Spinning Shell Inside the Bore of a Gun," Ballistic Research Laboratories Report No. 320 (AD 491839), Aberdeen Proving Ground, Maryland, February 1943.

2. L. H. Thomas, "The Motion of the Axis of a Spinning Shell Inside the Bore of a Gun," Ballistic Research Laboratories Report No. 544 (AD PB22102), Aberdeen Proving Ground, Maryland, May 1945.

3. H. P. Gay, "Notes on the Yawing Motion of a Projectile in the Bore," Ballistic Research Laboratories Report No. 2259 (AD 908456L), Aberdeen Proving Ground, Maryland, January 1973.

4. H. Goldstein, *Classical Mechanics*, Addison-Wesley Publishing Co., Inc., Cambridge, MA, March 1956.

5. H. Hertz, *J.f. Math.* (Crelle), Bd. 92 (1881).

6. C. V. Raman, "On Some Applications of Hertz's Theory of Impact," *Phys. Rev.* 15, 277-284 (1920).

7. A. E. H. Love, A Treatise on The Mathematical Theory of Elasticity, Dover Publications, N.Y. 1944.

8. F. P. Bowden and D. Taylor, The Friction and Lubrication of Solids, Clarendon Press, Oxford, G. B., 1950.

9. S. H. Chu and F. K. Soechting, Transverse motion of an accelerating shell, Technical Report 4314, Picatinny Arsenal, Dover, NJ, June, 1972. AD 894572L

10. S. H. Chu, Transverse Motion of 8 inch projectile, XM673, in the XM201, M2A2 Gun Tube, MK-16 and MCLG Gun, Technical Memorandum 2103, Picatinny Arsenal, Dover, NJ, August, 1973.

APPENDIX I

EQUIPMENT PERFORMANCE REPORT FOR 8" HOWITZER TUBE XM201 MECHANICAL FAILURE OF APRIL 15, 1974

The attached report describes the damage to the 8" Howitzer M110E2 as prepared by Mr. Hendrickson of MTD. A photograph (Figure A-1) of the damage to the tube is included. The stargage measurements and inspection data forms for the malfunction as prepared by P. Booth, J. McWilliams and R. Kane is also attached herewith.

Figures A-2 and A-3 give the pressure time records for the mechanical failure. Three tourmaline gages were located at 10", 29.65" and 42.90" (RFT), (i.e., at the spindle, chamber center and forcing cone locations). Figure A-2 shows a computer data reduction giving the difference between the measurements for the gages located at 10" and at 42.90" (RFT). Figure A-3 shows the computer plots for the three gages (channel 1 located at 10" (RFT), channel 2 at 29.65" (RFT)). The kicks in the PT trace are consistant with the interpretation that a pressure pulse propagated into the chamber, reflected off the rear face of the chamber and propagated back up the gun tube. In so doing, the pulse passed gage #1 once and gages #2 and #3 twice. The propagation velocity for the wave is 3000 ft/sec. At this velocity, the time required for a pressure wave to propagate from the position of the shell base at the beginning of the major damage to gage #1 is 5.0 msec. Since from Figure 5 the shell should arrive at the position of major damage at 9.5 msec after the pressure peak and since the pressure peak occurs at 13.0 msec in Figure A-3 (the zero time between Figures 5 and A-3 differs by about 7 msec), a pressure pulse associated with this event should arrive at strain gage #1 at 27.5 msec. The observed time is 27.25 msec. Essentially the same conclusion has already been reached by Ingo May of IBL in his consideration of the PT record of this event.

Mr. Hendricksen/dp/

EQUIPMENT PERFORMANCE REPORT

1. FROM Commander
U. S. Army Aberdeen Proving Ground
Aberdeen Proving Ground, Maryland 21005

2. OFFICE SYMBOL: STEAP-MT-A
3. DATE: 15 Apr 74
4. EPR NO. (K-2)-102

5. TO See Distribution List

6. USATECOM PROJ NO. 2-WE-200-110-004
RDT&E PROJ NO.
CONTRACT NO.

7. TEST TITLE Engineering Test of M110E2, SP, Full-Tracked, 8-Inch Weapon System

I. MAJOR ITEM DATA

8. MODEL Howitzer, SP, 8-Inch, M110E2	**9. SERIAL NO.**
10. QUANTITY 1	**11. LIFE PERIOD**
12. MFR.	**13. USA NO.**

II. PART DATA

14. NOMENCLATURE/DESCRIPTION Tube, 8-Inch, XM201, Serial No. 12

15. FSN	**16. MFR. PART NO.**
17. DRAWING NO.	**18. MFR.**
19. QUANTITY 1	**20. NEXT ASSEMBLY**
21. STD. GOVT. GRP.	**22. PART TEST LIFE** 142 Rounds

III. INCIDENT DATA

23. OBSERVED DURING	24. TEST ENVIRONMENT	25. INCIDENT CLASS	26. ACTION TAKEN
a. OPERATION	Firing with temperature conditioned ammunition	X a. DEFICIENCY	a. REPLACED
b. MAINTENANCE		b. SHORTCOMING	b. REPAIRED
X c. Firing		c. SUG. IMPROVEMENT	c. ADJUSTED
		d. OTHER	d. DISCONNECTED
			e. REMOVED
			X f. NONE Firing Suspended

27. DATE AND HOUR OF INCIDENT 11 Apr 74, 1100

IV. INCIDENT DESCRIPTION

28. DESCRIBE INCIDENT FULLY

1. While firing the M110E2 howitzer at 140 mils elevation, the M106 projectile broke up in bore damaging the tube. The projectile had an M557 fuze set superquick and was launched with an XM188 Zone 9 charge. Fragments from the projectile damaged a steel tower, velocity cage and coils, instrumentation bombproof, and camera tripod located in front of the muzzle. No personnel injuries resulted.

2. Damage to the weapon consisted of moderate to heavy land damage between 245 and 250 inches from the rear face of the tube (See inclosed photograph.). Star-gage readings of the groove diameters also indicated a slight bulge in this same region.

3. Ammunition involved in the malfunction included the following:

a. Projectile, 8-Inch, M106, HE, Lot No. 10P 17-75

b. Fuze, PD, M557, Lot No. MA 19-3

c. Charge, Propelling, 8-Inch, XM188, Lot No. IND-E-116-74

DEFICIENCIES AND SHORTCOMINGS ARE SUBJECT TO RECLASSIFICATION

29. DEFECTIVE MATERIAL SENT TO: Held at APG

30. NAME, TITLE & TEL EXT OF PREPARER
Ronald Hendricksen
RONALD HENDRICKSEN
Test Director
FA, AA, & Sp Ammo Sec,
Arty Ammo Br, Arty Div, MTD, Ext. 2267

31. FOR THE COMMANDER
H. B. Anderson
H. B. ANDERSON
Acting Chief, Artillery Division
Materiel Testing Directorate

STE Form 1025, 10 Feb 70 EDITION OF 18 DEC 69 MAY BE USED TECR 70-23

Each of the ammunition components had been subjected to sequential rough handling tests at -70°F prior to firing. The specific rough handling conditions for each component were as follows:

(1) The projectile had been dropped 7-feet unpackaged to impact a steel plate base first with the axis 45° from the vertical. Following this, the projectile was subjected to a bounce test with the projectile in its normal vertical orientation.

(2) The fuze was subjected to successive 7-foot drops in six different orientations while in its normal packaging. Following this, the packaged fuze was bounce tested with the fuze horizontal. Finally, the fuze was affixed to a 105-mm M1 shell and dropped 5 feet so as to strike nose down with the shell 45° from the vertical.

(3) The packaged propelling charge was dropped 7 feet so as the strike the steel plate with the base end of the charge down and the charge 45° from the vertical. Then the charge was bounce tested with the charge vertical, base charge end down. Finally, the unpackaged charge was dropped horizontally and base down from a height of 5 feet.

Following each phase of the rough handling sequence, each component was visually inspected to establish if the items were safe to continue testing. At the end of the sequence, all components were visually inspected. In addition, the projectiles and fuzes were x-rayed to examine for internal damage. X rays of the projectile involved in the malfunction showed cracks in the TNT filler between the forward bourrelet and the nose of the shell. The fuze showed slight visual damage to the nose of the fuze but no internal damage. The propelling charge suffered a seam rip after two of the planned five drops. This rip was mended before firing, but no further drops were conducted.

4. At the time of firing, the ambient air temperature was 53°F. All ammunition components were temperature conditioned to -70°F. The malfunction occurred on the eighth round of the day and 11th test round of the safety test. Comparison data for the malfunctioned round and the previous test rounds combined are provided in the following table.

Comparison Data for M110E2 Safety Test Firings - Malfunctioning Versus Nonmalfunctioning Rounds

Type of Rd	No. of Rds	Preconditioned	Firing Temperature, °F	Seating, In	Avg Peak Pressure, KSI	Avg Vel fps	Avg Recoil, In	Avg Rg, M
Malfunction	1	RH	-70	44 1/2	36.1	NA	49 1/2	NA
Test	10	RH	-70	44 3/8	36.7	2457	49 7/8	9532

. There were two small kicks in the PT trace 17 milliseconds after the peak pressure was reached, but otherwise the trace was normal and similar to previous test rounds.

5. Investigation of the incident is continuing.

1 Incl
Photograph

DISTRIBUTION LIST

Equipment Performance Report
Engineering Test of M110E2, SP, Full-Tracked, 8-Inch Weapon System
TECOM Project No. 2-WE-200-110-004

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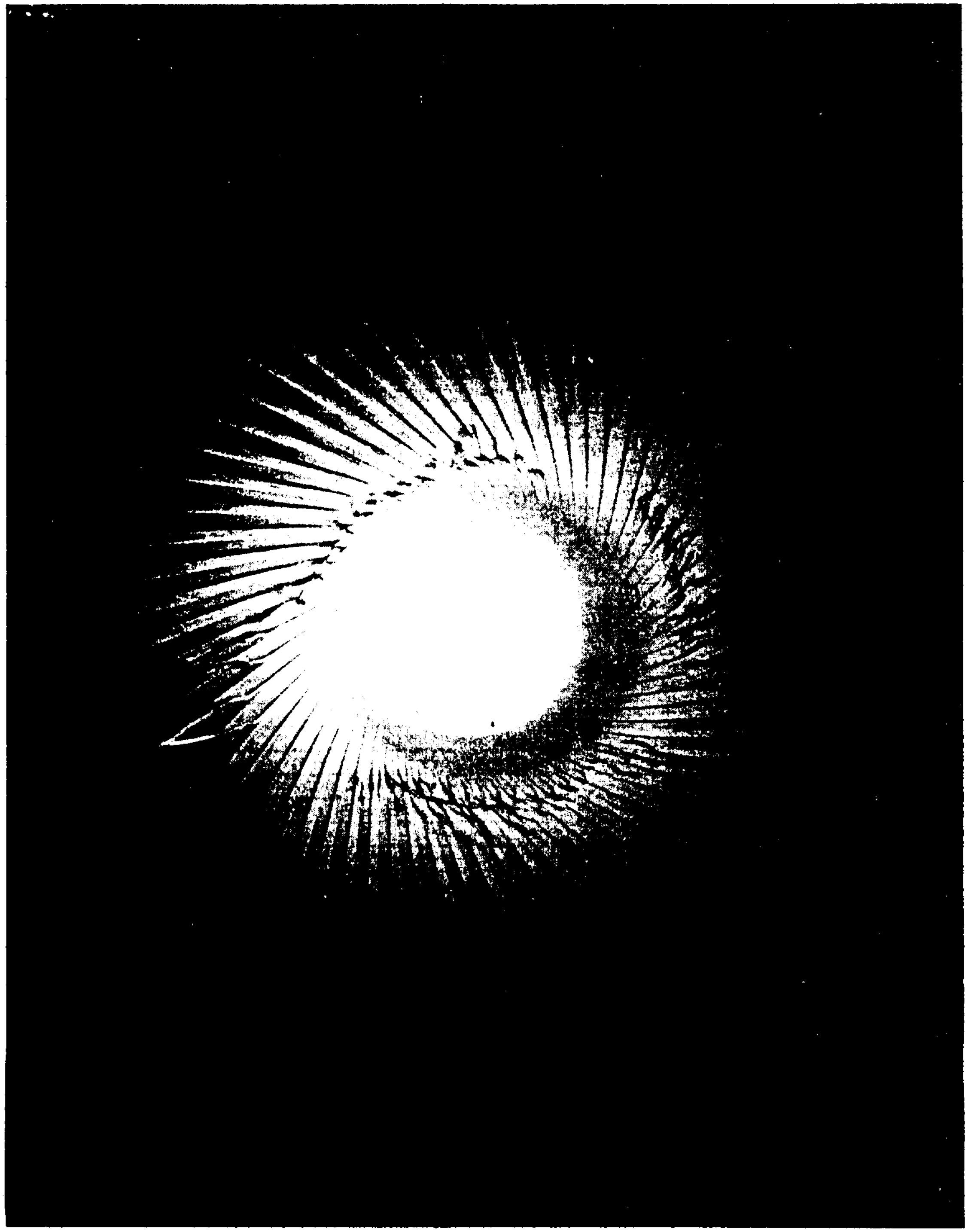


Figure A-1. Photograph of damage to the 8" XM201 Howitzer Tube looking from the Muzzle end.

MULTIPLE STARGAGE MEASUREMENT & INSPECTION DATA FORM

CASTING NUMBER	X
MANUFACTURER	WVT. ARS.
PROOF OFFICER	Mr. HENDRICKSEN W.O. 305-96401-02
MODEL	XM201 MOD.
NUMBER OF ROUNDS	142
NUMBER	12
FIRING STATUS (Check One)	AFTER ✓ / BEFORE
	8.00" HOW TUBE
	8.00" HOWITZER
DATE OF GAUGING	11

8.00" Howitzer Tube XM201 MOD.

Main bore - 46.12" to 315.86"

Distance (Inches) from - REAR FACE OF BREECH	MUZZLE FACE	REAR FACE OF TUBE ✓	Gage Read. indicated in 1/1000 of an inch — LANDS 8.000" Basic Dia. Vert.	LANDS Hor.	GROOVES 8.140" Basic Dia. Vert.	GROOVES Hor.
322.35	.61	315.25	+.006	+.005	.000	.000
322.10	.86	315.00	4	3	0	0
317.10	5.86	310.00	2	2	0	0
312.10	10.86	305.00	3	2	0	0
307.10	15.86	300.00	3	2	0	0
302.10	20.86	295.00	3	2	0	0
297.10	25.86	290.00	3	3	0	+.001
292.10	30.86	285.00	3	4	+.001	1
287.10	35.86	280.00	5	5	1	1
282.10	40.86	275.00	9	7	2	1
277.10	45.86	270.00	10	7	4	1
272.10	50.86	265.00	10	7	4	1
267.10	55.86	260.00	9	7	5	1
262.10	60.86	255.00	16	13	7	1
257.10	65.86	250.00	20	4	3	.000
252.10	70.86	245.00	2	5	.000	0
247.10	75.86	240.00	2	5	0	0
242.10	80.86	235.00	2	4	0	0
237.10	85.86	230.00	3	3	0	0
232.10	90.86	225.00	3	3	0	0
227.10	95.86	220.00	3	3	0	0
222.10	100.86	215.00	3	3	0	0
217.10	105.86	210.00	3	3	0	0
212.10	110.86	205.00	3	3	0	0
207.10	115.86	200.00	3	3	0	0
202.10	120.86	195.00	3	4	0	0
197.10	125.86	190.00	3	4	0	0
192.10	130.86	185.00	3	4	0	0
187.10	135.86	180.00	3	4	0	0
182.10	140.86	175.00	3	4	0	0
177.10	145.86	170.00	3	4	0	0
172.10	150.86	165.00	3	4	0	0
167.10	155.86	160.00	3	4	-.001	0
162.10	160.86	155.00	3	4	1	0
157.10	165.86	150.00	3	3	1	0
152.10	170.86	145.00	3	3	1	0
147.10	175.86	140.00	2	3	1	-.001
142.10	180.86	135.00	2	3	2	1
137.10	185.86	130.00	2	4	2	1
132.10	190.86	125.00	3	4	2	1
127.10	195.86	120.00	3	4	2	1
122.10	200.86	115.00	3	5	2	1
117.10	205.86	110.00	4	5	2	1
112.10	210.86	105.00	4	5	2	1
107.10	215.86	100.00	4	5	2	1
102.10	220.86	95.00	4	5	2	1
97.10	225.86	90.00	3	4	2	2
92.10	230.86	85.00	2	3	2	2
87.10	235.86	80.00	1	2	2	2
82.10	240.86	75.00	1	1	2	2
77.25	245.71	70.00	1	1	1	2
72.10	250.86	65.00	1	1	1	2
69.25	253.71	62.15	2	1	1	2
67.10	255.86	60.00	3	2	.000	2
61.25	261.71	54.15	.003	+.002	-.000	[illegible]
259.[illegible]	(MAX READING)		+.026	+.017	+.007	+.0[illegible]1

Bore Contin[illegible] on page 2

8.00" Howitzer Tube XM201 MOD.	Chamber 1.50" to 46.15

8" TUBE, #14 XM201 MOD.

11 APRIL 1974 142 ROUNDS

FOR: MR. HENRICKSON WO 305-95401-02

DISTANCE (Inches) FROM					GAUGE MEASUREMENTS INDICATED IN 1/1000 OF AN INCH					
					~~VERTICAL~~ Y			~~HORIZONTAL~~ X		
REAR FACE OF BREECH	MUZZLE FACE	REAR FACE OF TUBE	BASIC DIAMETER	ZERO	GAUGE READING	ACTUAL DIAMETER	DIFFERENCE	GAUGE READING	ACTUAL DIAMETER	DIFFERENCE
50.10	272.86	43.00	8.300	8.300	-.005	8.295	-.005	-.005	8.295	-.005
49.10	273.86	42.00	8.305		+.002	8.302	-.003	+.002	8.302	-.003
47.10	275.86	40.00	8.314		11	311	3	10	.310	4
45.10	277.86	38.00	8.323		19	319	4	19	.319	4
43.10	279.86	36.00	8.332		28	328	4	28	.328	4
41.10	281.86	34.00	8.341		37	337	4	37	.337	4
39.10	283.86	32.00	8.350		46	346	4	46	.346	4
37.10	285.86	30.00	8.359		55	355	4	55	.355	4
35.10	287.86	28.00	8.368		64	364	4	64	.364	4
33.10	289.86	26.00	8.377		75	375	2	75	.375	2
31.10	291.86	24.00	8.386		84	384	2	84	.384	2
29.10	293.86	22.00	8.395		94	394	1	94	.394	1
27.10	295.86	20.00	8.404		.104	.404	.000	104	.404	.000
25.10	297.86	18.00	8.413		.111	411	-.002	111	411	-.002
23.10	299.86	16.00	8.422		.119	419	3	119	.419	[illegible]
21.10	301.86	14.00	8.431		.128	428	3	129	429	2
19.10	303.86	12.00	8.440		.138	438	2	138	.438	2
17.10	305.86	10.00	8.449		.147	447	2	148	.448	1
15.10	307.86	8.00	8.458		.156	456	2	158	.458	.00[illegible]
13.10	309.86	6.00	8.467		.165	465	2	167	467	0
11.10	311.86	4.00	8.476		.175	475	1	176	476	0
9.10	313.86	2.00	8.485		.185	485	.000	186	486	+.001
8.70	314.26	1.60	8.487		+.188	8.488	+.001	+.184	8.489	+.002

SPECIAL MEASUREMENTS

	BASIC	ACTUAL		BASIC	ACTUAL
TOTAL LENGTH OF GUN	322.96"	—	ROTATION OF TUBE AT BREECH	—	—
TOTAL LENGTH OF TUBE	315.86"	315.85"	MOVEMENT OF TUBE AT BREECH	—	—
DEPTH OF BREECH RECESS	7.10"	—	NUMBER OF LANDS AND GROOVES	64	64

Inspection Remarks: Areas from 0.00 to 1.50", 43.10" to 46.15" and from 315.25" to 315.86" were not measured.

STAMPED PREV. APG.	STARGAUGED AND INSPECTED BY BOOTH	REVIEWED BY
RODMAN McWILLIAMS	TIME	COMPILATOR
RECORDER KANE	PLACE 525	[illegible] BY

-DC Form Jun (

MULTIPLE STARGAGE MEASUREMENT & INSPECTION DATA FORM

- 8.00" HOW. TUBE
- 8.00" HOWITZER
- CASTING NUMBER: X
- MANUFACTURER: WVT ARS.
- MODEL: XM201 MOD
- NUMBER: 12
- PROOF OFFICER: MR. HENRICKSON, WD 305-96401-02
- NUMBER OF ROUNDS: 142
- FIRING STATUS (Check One): BEFORE / AFTER ✓
- DATE OF GAUGING: 11 APRIL 1974

8.00" Howitzer Tube XM201 MOD.

Main Bore - 46.12" to 315.86

Distance (inches) from - REAR FACE OF BREECH	MUZZLE FACE	REAR FACE OF TUBE	Gage Meas. Indicated in 1/1000 of an inch — LANDS 8.000" Basic Dia. Vert.	LANDS Hor.	GROOVES 8.140" Basic Dia. Vert.	GROOVES Hor.
56.25	266.71	49.15	+.001	+.001	-.001	-.002
55.25	267.71	48.15	1	2	1	2
54.25	268.71	47.15	2	3	1	2
54.00	268.96	46.90	2	4	1	1
53.75	269.21	46.65	3	4	1	1
53.50	269.46	46.40	4	4	1	1
53.35	269.61	46.25	+.004	+.004	-.001	-.001
			Vert.	HoR.		
Pullover Meas.		46.40	8.005"	8.003"	Estimated Remaining Accuracy Life	%

SPECIAL MEASUREMENTS		Basic	Actual
Opening between breech bushing and rear face of tube	Top	X	X
	Bottom		
	Right		
	Left		
Rotation of breech bushing at R.F.T.			

	Trammel Distance Established B.F.	Rds.
Movement of Hoop "A" over Tube	2.000	1.987
Movement of Hoop "B" over Tube	2.005	[illegible]
Movement of Hoop "C" over Tube	2.006	2.029

Borescope Remarks: (Chrome Plated Chamber and Bore)

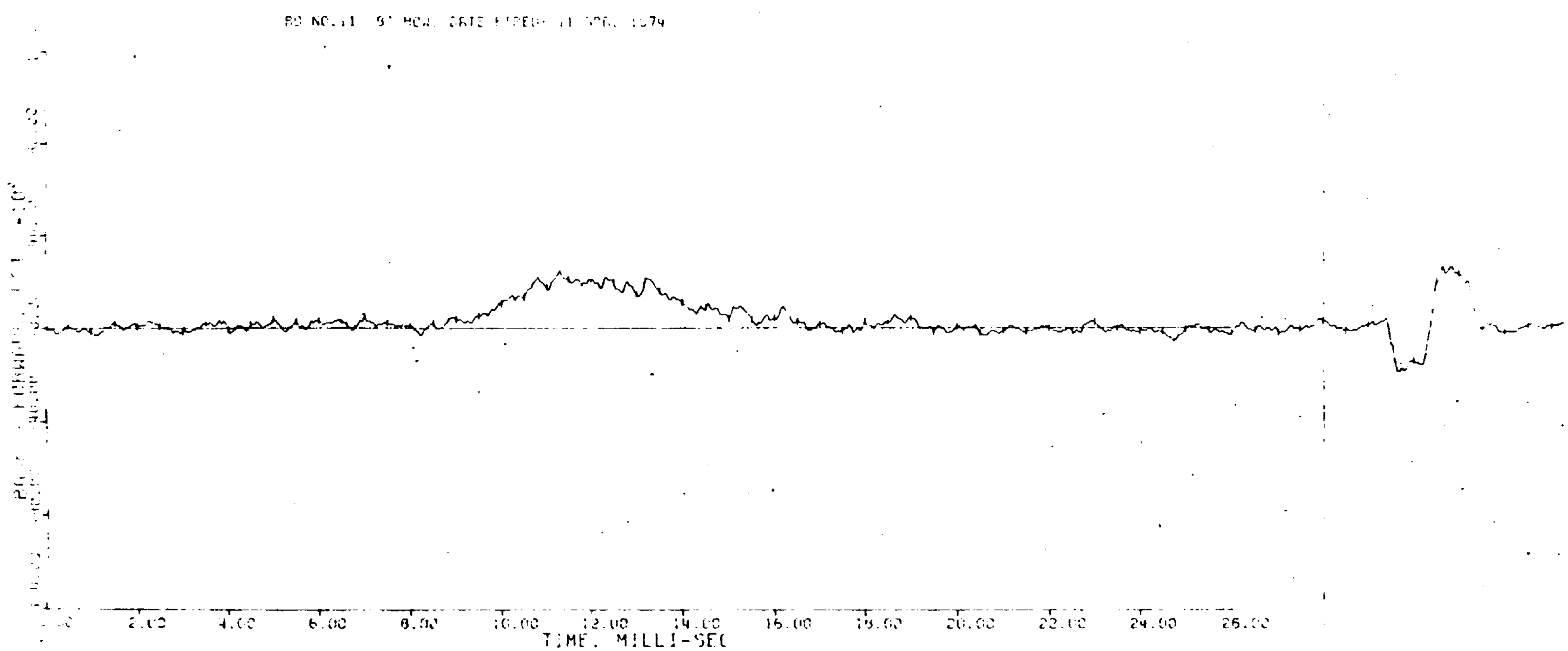
Several light longitudinal scratches with light powder fouling, carbon and other deposits thru-out chamber. Light heat checking beginning in centering cylinder and becoming moderate at origin of rifling and extending forward to 60.00" in the grooves and to 140" on lands from rear face of tube (RFT). Chrome lightly chipped on edges of lands encircling commencement of rifling and vicinity. Light powder foulings, carbon and other deposits thru-out bore. Driving and non-driving edge of one land sheared away between 185.00" and 186.75" from (RFT) at 1:00 o'clock. Heavy to moderate damage to lands encircling bore between 249.75" and 258.50" from (RFT). Damages consist of lands flattened and compressed with highly polished brass deposits at 249.75" from (RFT). From this point forward edges of lands have split away and sprung out from the base of the land for a distance of approximately 8.00". Lands are still intact at the flattened area at 259.75" from (RFT). Four lands at 2:00 o'clock, 3 lands at 10:00 o'clock and 5 lands at 8:00 o'clock thru-out this area have not been damaged. Moderate to light similar damage to 8 lands between 273.75" and 280.75" from (RFT) at 2:00 o'clock.

Due to damages in bore tube is "HAZARDOUS" in accordance with MTD 750-1, Sec 2, Par. b.

A series of photos were taken thru-out damaged areas.

Gaged By: P. Booth
J. McWilliams
R. Kane

STEAP-DS Form 106, 17 Jun 64 (Part I)



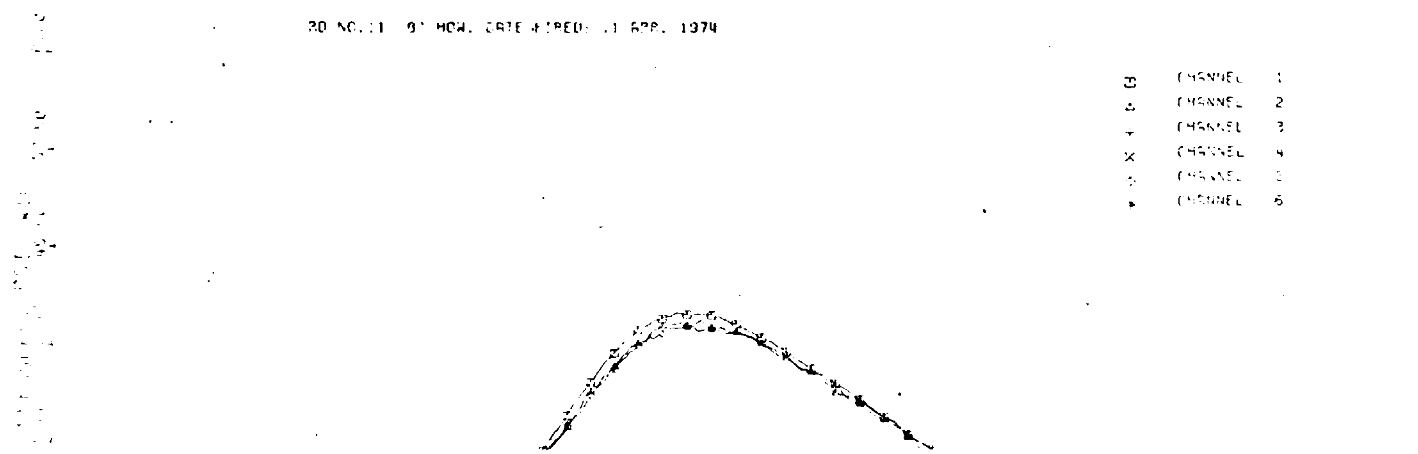


Figure A-2. **Plot of the pressure difference between strain gage #1 at the spindle and strain gage #3 at the forcing cone as a function of time.**

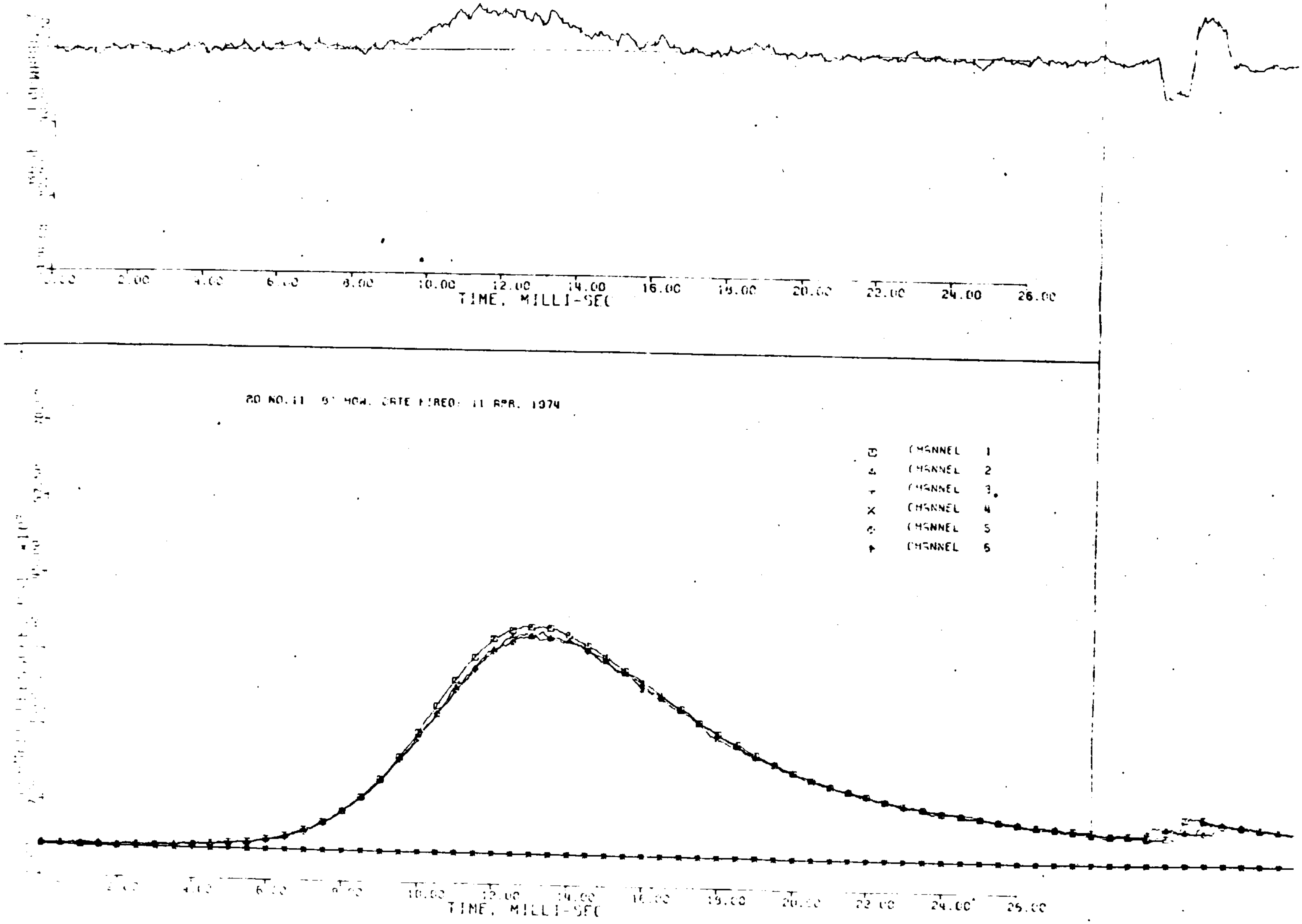


Figure A-3. Plot of the pressure vs. time for three tourmaline strain gages located at the spindle (#1, 10" RFT), middle of chamber (#2, 29.65" RFT) and at the forcing cone (#3, 42.90" RFT).

LIST OF SYMBOLS

a	distance between the center of the bourrelet and rotating band, also, radius of gun tube wall area deflected by shell impact at end of impact ($t = t_2$)	m
b	radius of shell at bourrelet	m
b'	radius of shell at rotating band	m
b_1	half thickness of gun tube wall	m
c	clearance between bourrelet and bore of gun for zero yaw	m
c_s	velocity of sound in shell	m/sec
e	coefficient of restitution	dimensionless
k_a	radius of gyration about shell axis	m
ℓ	distance from CG to the point on shell's axis at center of the rotating band	m
ℓ_b	bourrelet radius of curvature (in cross-section) at point of contact with lands of gun tube	m
m	mass of shell	Kg
m_1, m_2	mass of balls	kg
n	twist of the rifling	(calibers/turn)
p_b	base pressure	newtons/m^2
p_c	chamber pressure	newtons/m^2
q_1, q_2	Young's modulus (gun tube, shell, respectively)	newtons/m^2
q_i, q_1, q_2, q_3	generalized Lagrangian coordinates	($Q_i \times q_i$ have dimensions of energy
r	radius of gun tube bore	m
r_1, r_2	radius of balls	m
s	distance traveled by shell along z coordinate	m

LIST OF SYMBOLS

t	time	sec
t_s	time from onset of shell motion to shell exit	sec
t_1	time from first contact on impact between bourrelet and gun bore to maximum yaw δ_{max}	sec
t_2	time from first contact on impact between bourrelet and gun bore to end of impact contact	sec
v	velocity of bourrelet at impact with bore	m/sec
x	x-component of rectiliner coordinates fixed with reference to the gun tube; z-component co-linear with gun tube axis	m
x_s	x-component of rectilinear coordinates fixed with reference to the shell; z-component co-linear with shell axis	m
y	y-component of rectilinear coordinates fixed with reference to the gun tube; z-component co-linear with gun tube axis	m
y_s	y-component of rectilinear coordinates fixed with reference to the shell; z-component co-linear with shell axis	m
z	z-component of rectilinear coordinates; co-linear with gun tube axis	m
z_s	z-component of rectilinear coordinates; co-linear with shell axis	m
A	axial moment of inertia	kg m^2
B	transverse moment of inertia	kg m^2
C	the point on the shell's axis at the center of the rotating band	-
C_1	constant of integration	(dimentionless)
C_2	constant of integration	kg m^2
E_s	translational kinetic energy of shell	Joules

LIST OF SYMBOLS

E_ψ	spin kinetic energy of shell	Joules
I	moment of inertia about the point on shell's axis at center of the rotating band	kg m^2
I'	constant (see Eq. 27)	(dimensionless)
K_c, K_r	force constant (Hooke's law)	Newtons
L	Lagrangian	Joules
L_e	gun tube length (from forcing cone to muzzle)	m
Q_i	components of the generalized Lagrangian force	($Q_i x q_i$ have dimensions of energy)
Q_1, Q_2, Q_3	components of the generalized Lagrangian force	($Q_i x q_i$ have dimensions of energy)
S	angular acceleration in yaw per sin δ (see Eq. 29)	radians/sec^2
T	rotational kinetic energy, also, torque acting on the rotating band due to the <u>rifling</u> (excluding impact and reactions force torques) of the gun tube	Joules Newton-m
$\overline{T}_y$	time averaged torque for y-components due to impacts	Newton-m
$\overline{T}_z$	time averaged torque for z-components due to impacts	Newton-m
T'	total torque on shell acting in z-direction ($T' = Q_3$)	Newton-m
V	potential energy of the shell in a coordinate system moving with the shell (accelerating coordinate system)	Joules
X	x-component of impact force acting on bourrelet	Newtons
X'	x-component of impact reaction force acting on rotating band	Newtons
X_i^*	spin up forces of lands against rotating band	Newtons
Y	y-component of impact force action on bourrelet	Newtons

LIST OF SYMBOLS

Y'	y-component of impact reaction force acting on rotating band	newtons
Z	z-component of impact force acting on bourrelet	newtons
Z'	z-component of impact reaction force acting on rotating band	newtons
α	shell spin rate as given by Thomas	radius/sec
δ	yaw angle, inclination of the shell axis from gun tube bore axis	(radians)
δ_c	angle between the shell axis and a line passing through the point C and the CG of the shell	(radians)
δ_o	initial yaw angle of shell as loaded in gun chamber	(radians)
δ_{max}	maximum yaw of shell during impact between bourrelet and gun bore	(radians)
$\dot{\delta}_o$	angular velocity at $\delta = o$ (zero yaw)	radians/sec
γ	constant (see Eq. 77)	(dimensionless)
ε	kinetic energy in the yawing (δ-component only) motion of the shell	Joules
ε_o	initial (first impact between bourrelet and gun bore)kinetic energy in the yawing (δ-component only) motion of the shell	Joules
Θ	total angle swept out by the shell in the δ direction irrespective of sign	(radians)
κ	Raman's coefficient	(dimensionless)
λ	wave length of flexural waves in shell	m
μ	coefficient of friction between bourrelet and gun tube	(dimensionless)
ν	coefficient of friction between rotating band and gun tube	(dimensionless)
ξ	ratio of lands area to total bore area in gun tube	(dimensionless)

LIST OF SYMBOLS

ρ_1, ρ_2	density (gun tube, shell, respectively)	Kg/m^3
σ_1, σ_2	poisson's ratio (gun tube, shell, respectively)	(dimensionless)
τ_e	time lag between impact on bourrelet and reaction force at rotating band	sec
ϕ	azimuth of the shell about the gun tube bore axis	(radians)
ψ	rotation angle of the shell about its own axis	(radians)
ω_x, ω_y, ω_z	components of angular velocity about axes, x, y, z	sec^{-1}
Δt	time between impacts of bourrelet	sec
Δt_o	time from onset of shell motion to first bourrelet impact	sec
$\Delta\phi$	angle of precession during bourrelet impact	sec
Ω	shell spin rate about shell axis	radians/sec
Ω'	constant (see Eq. 28)	radians/sec

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A review of the reports listed below has been completed and it has been determined that the distribution limitation can be removed and the Statement "A" be applied:

REPORT	TITLE	DATE	AD NO
BRL Report No. 1406	BRL Survey Of The Army Caseless Ammunition Program.	Jun 68	844887L
BRL Report No. 1756	The Effect Of Muzzle Jet Asymmetry On Projectile Motion.	Jan 75	B002159L
BRL Report No. 1758	Wind Tunnel Magnus Tests Of Cylind-rical And Boattail Army-Navy Spinner Projectiles With Smooth Surface And 20MM Equivalent Engraving (Rifling Grooves).	Feb 75	B002628L
BRL Report No. 1793	The Influence Of Muzzle Gasdyna-mics Upon The Trajectory Of Fin-Stabilized Projectiles.	Jun 75	B005379L
BRL Report No. 1899	Plane Shock - Thermal Layer Interaction.	Jul 76	B013138L
BRL Report No. 1900	The Effect Of Wind On Flat-Fire Trajectories.	Aug 76	B012872L
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BRL Memorandum Report No. 1953	A Comparative Evaluation Of The 7.62MM And 5.56MM, G-3 Assault Rifles.	Jan 69	849576
BRL Memorandum Report No. 2211	Optimum Projectile Shape For Improving Ammunition.	Aug 72	906481L
BRL Memorandum Report No. 2215	A Computer Program To Calculate The Physical Properties Of A System Of Coaxial Bodies Of Revolution.	Aug 72	904378L
BRL Memorandum Report No. 2225	Determination Of Muzzle Velocity Changes Due To Nonstandard Propellant Temperature Using An Interior Ballistic Computer Simulation.	Sep 72	905714L
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BRL Memorandum Report No. 2496	Experiments On Wake Optical Properties.	Jul 75	B005619L
BRL Memorandum Report No. 2501	Preliminary Surveys Of The Three Dimensional Boundary Layer On A Yawed, Spinning Body Of Revolution.	Jul 75	B005829L
BRL Memorandum Report No. 2536	The Effect Of A Sub-Caliber Cylinder After-Body On The Behavior Of Spin-Stabilized Projectiles.	Sep 75	B008388L
BRL Memorandum Report No. 2563	Characteristics Of The Sidewall And Floor Boundary Layers In BRL Supersonic Wind Tunnel No. 1.	Dec 75	B008566L
BRL Memorandum Report No. 2573	Three-Dimensional Boundary Layer Research As Applied To The Magnus Effect On Spinning Projectiles.	Dec 75	B008821L
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BRL Technical Note No. 1691	Talk To Cadets Of The United States Military Academy, April 1968, Special Purpose Individual Weapon (SPIW).	Apr 68	390618

FOR THE DIRECTOR:

Vernon J. Wyatt

VERNON J. WYATT
Chief
Technical Support Division